Fall due to g with quadratic dissipation.

These graphs (third page) compare several simple ODE numerical solutions (as opposed to Runge-Kutta, etc.). The ODE is taken from Eisberg and Lerner's skydiver.(1) The skydiver formula assumes the usual simplifications, for example, g is constant, and the dissipation coefficient also, *i.e.* Reynolds number is between 10^3 and 10^5, and no other variation, which is, obviously, not true, but irrelevant to the comparison.

The comparison is shown by fitting to the analytic solution. The next page defines the criteria of comparison. Most useful is kaleidagraph's Chi Square, which is seen simply the difference between each datum and the corresponding analytic value squared and then all summed.

Included are the C++ source codes (in PDF), so one may verify the solutions.
John Denker wrote the shell for me; I wrote the numericals and modified the shell appropriately.

Some appropriate articles regarding the approximations:

http://scitation.aip.org/content/aapt/journal/ajp/49/5/10.1119/1.12478

http://young.physics.ucsc.edu/115/leapfrog.pdf

http://www.av8n.com/physics/symplectic-integrator.htm

http://www.cems.uvm.edu/~tlakoba/math337/notes_1.pdf

bc

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# D.6 Pearson's R and Chi Square Equations

**Pearson's R (Least Squares curve fits)**

$$R = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{N\sum x_i^2 - (\sum x_i)^2}\sqrt{N\sum y_i^2 - (\sum y_i)^2}}$$

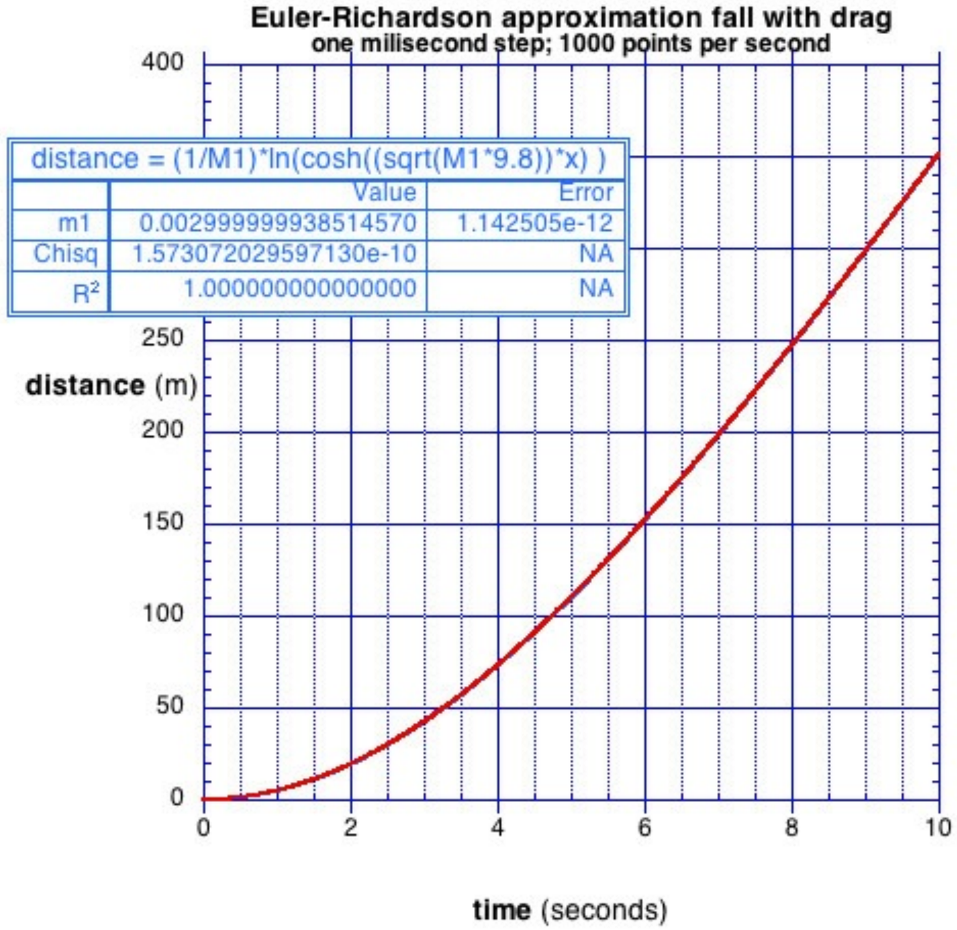
**Pearson's R (General curve fit)**

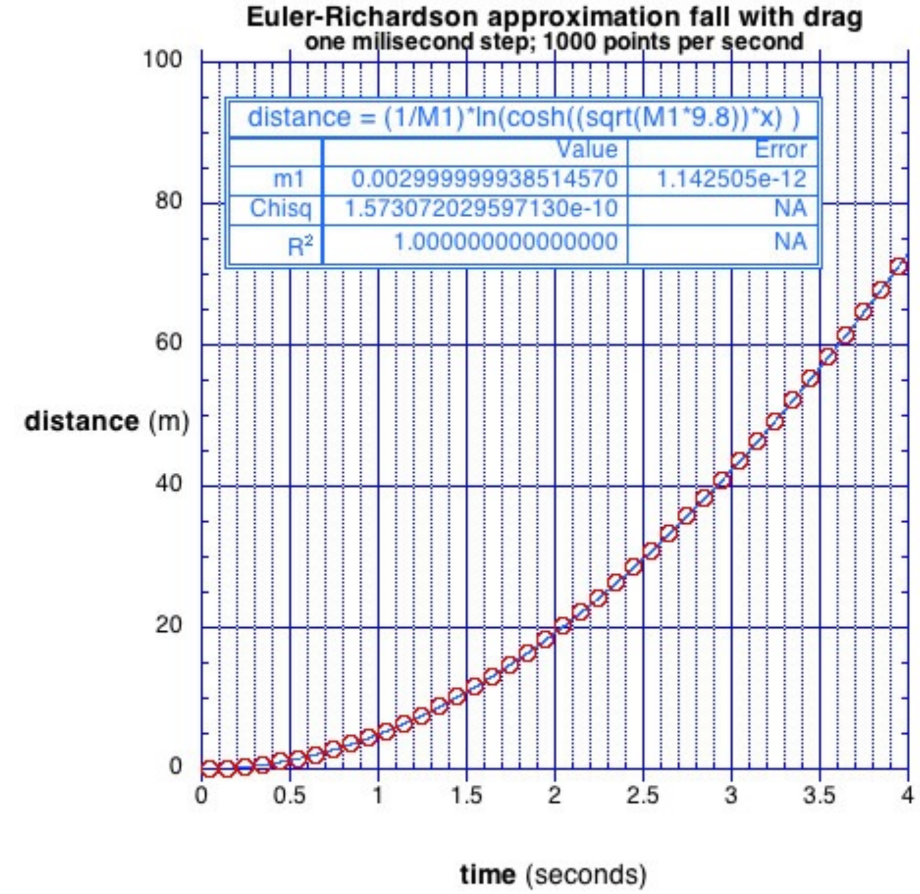
$$R = \sqrt{1 - \frac{\chi^2}{\sum_i \sigma_i (y_i - \bar{y})^2}}$$

**Chi Square**

$$\chi^2 = \sum_i \left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2$$

The sigma is the weighting factor, which is, in all cases, one.


Euler-Richardson approximation fall with drag
one milisecond step; 1000 points per second
distance = (1/M1)*ln(cosh((sqrt(M1*9.8))*x) )
Value Error
m1 0.002999999938514570 1.142505e-12
Chisq 1.573072029597130e-10 NA
R² 1.000000000000000 NA
distance (m)
time (seconds)


Euler-Richardson approximation fall with drag
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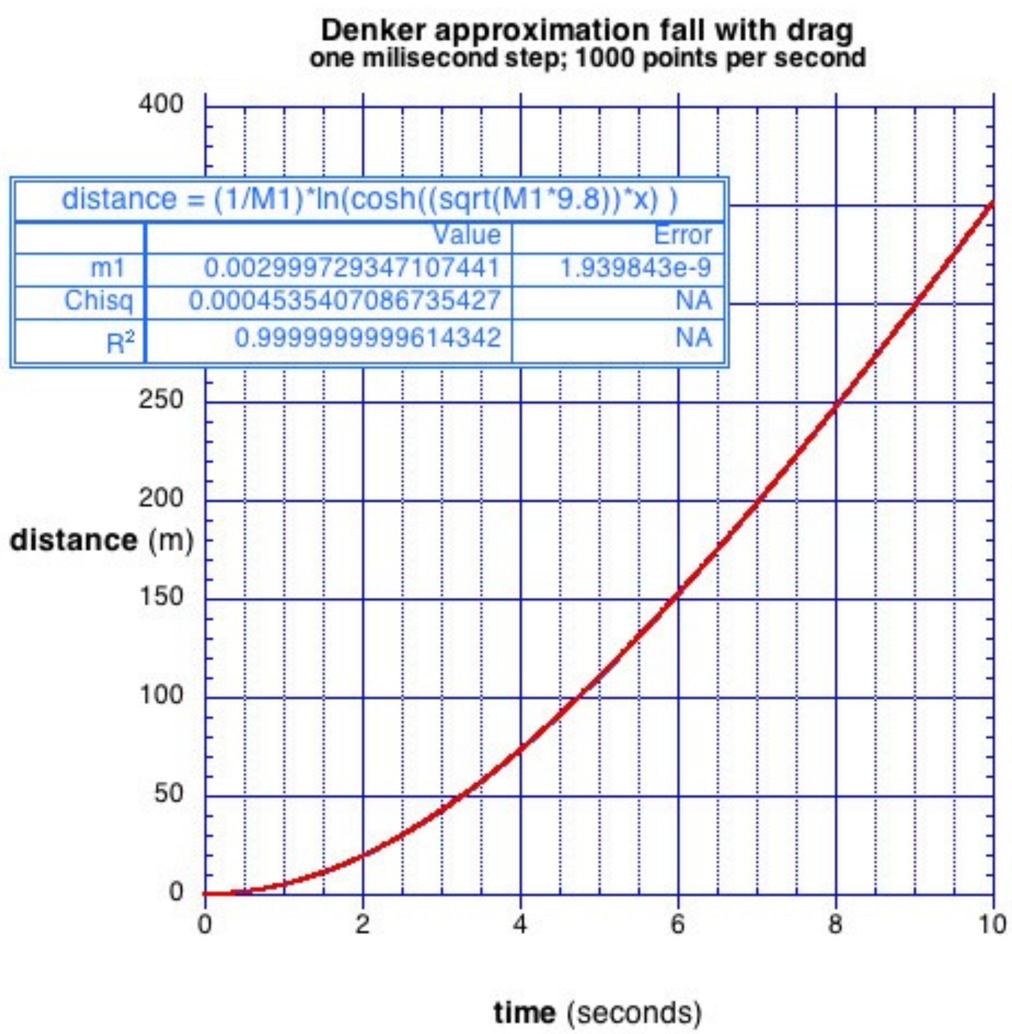
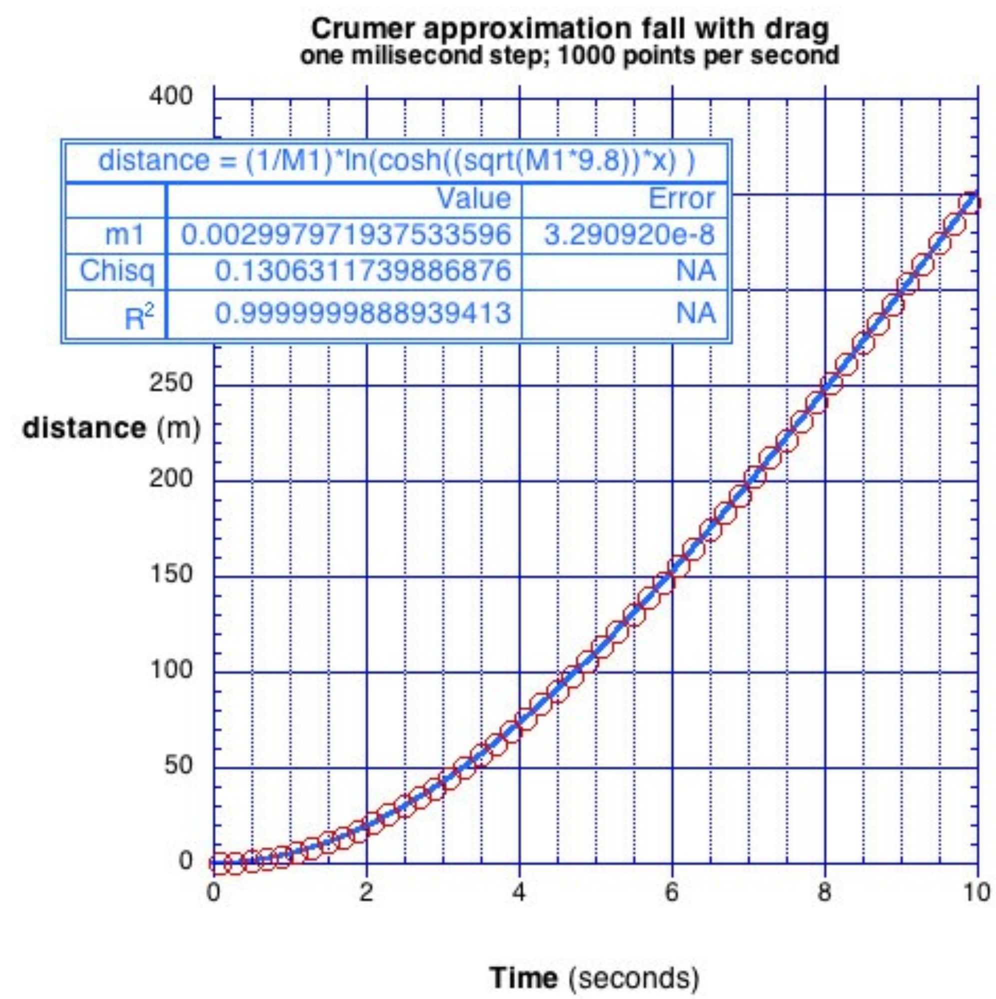
Value Error
m1 0.002999999938514570 1.142505e-12
Chisq 1.573072029597130e-10 NA
R² 1.000000000000000 NA
distance (m)
time (seconds)


Denker approximation fall with drag
one milisecond step; 1000 points per second
distance = (1/M1)*ln(cosh((sqrt(M1*9.8))*x) )
Value Error
m1 0.002999729347107441 1.939843e-9
Chisq 0.0004535407086735427 NA
R² 0.9999999999614342 NA
distance (m)
time (seconds)


Crumer approximation fall with drag
one milisecond step; 1000 points per second
distance = (1/M1)*ln(cosh((sqrt(M1*9.8))*x) )
Value Error
m1 0.002997971937533596 3.290920e-8
Chisq 0.1306311739886876 NA
R² 0.9999999888939413 NA
distance (m)
Time (seconds)