Graphs of the Harmonically Driven Linearly Damped Spring Oscillator Responses as a Function of their Masses

The equation used is (3.59) of Thornton | Marion[1]and the Qs are calculated using equation (3.64)[2].

I have "put" them in response to both questions discussed on the physics list (phys-l), during personal communications, and published in the Horological Science Newsletter.

The graphs show, contrary to some person's belief, that

One: The maximum amplitude does not occur at the undamped natural frequency, and
Two: Increasing mass results in decreased maximum amplitude.

Note, however, with increasing mass (and resulting Q) the maximum frequency is asymptotic to the natural frequency, and the maximum amplitude is also asymptotic.

Since periodic forcing modeling, using either a Laplace transformation or delta function, results similarly to harmonic forcing[3], I suspect a similar result, which is horologically interesting.

Bernard Cleyet, PhD

http://www.cleyet.org/

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[1] p. 118, 5th ed. $A_\omega = \dfrac{F_0 / m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2 (b/m)^2}}$

[2] p. 121, op. cit. $Q = m\dfrac{\sqrt{\omega_0^2 - \dfrac{b^2}{2m^2}}}{b}$

[3] Baker and Blackburn, The Pendulum, pp. 37 ff.

p.s. T | M's graph[4] of the amplitude (D) as a function of the driving frequency for various Q's is quite misleading. It shows D increasing with increasing Q. As I've shown this is not generally true. This error is very common. The amplitude (A) in equation (3.59) is F_0 / m, not just the driving amplitude, which A implies. I've made this explicit in the foot note 1 above. Of course, if the increased Q is due to a decreased damping coefficient, then the D (amplitude) will increase.

[4] p. 121 op. cit.