An Elucidation of Three Dissipation Contributions Limiting Pendulum Quality, "Q", Using a "Grand Mother" Clock

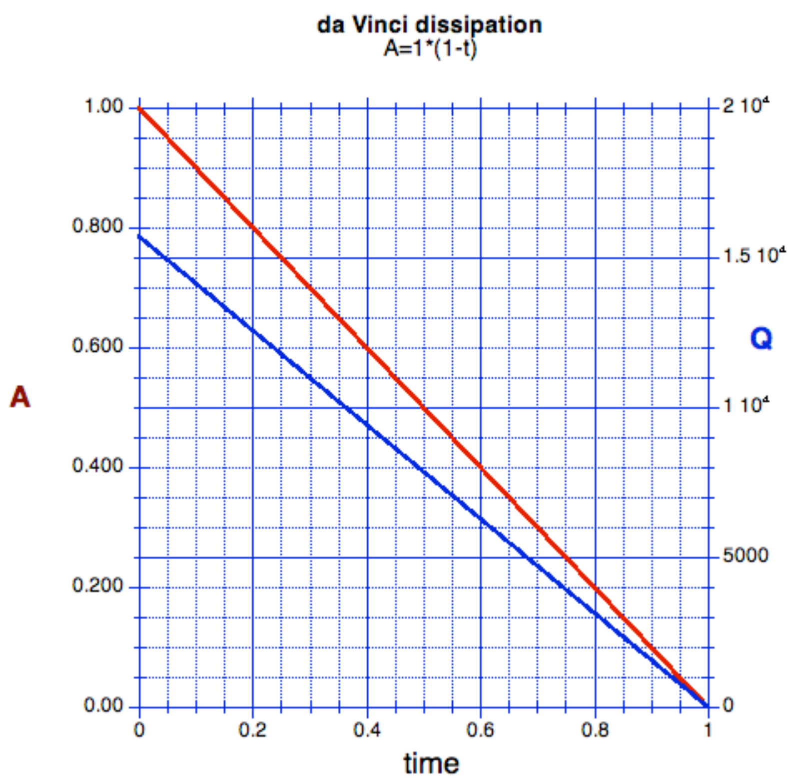
Part 1

In a cursory perusal of the HSN archive I found only one case describing the attachment of a "sail" to a clock's pendulum to reduce its Q. Furthermore, I didn't find an article which described the measurement of pendulum Q, continuously [1]. To correct this, using an Emperor model 100 clock, I have measured the quasi-free decay (drive weight removed) with and without an attached sail, the driven decay to equilibrium from the maximum displacement (limited by the clock's case), also with and without a sail, the free [escapement anchor staff (with included crutch) removed] decay of the pendulum from the maximum amplitude, again with and without an attached sail, finally, some of the preceding with an added lab. disk weight (0.5 kg) glued to the pendulum's bob. By doing this I will show the effect of three dissipation types on pendula.

First I present three dissipation models as an aid in the interpretation of acquired Q and amplitude data. They are constant friction (da Vinci[2] or sliding dry friction), linear (viscous and hysteric), and quadratic drag. Sliding or constant friction, first systematically studied by L. da Vinci, is independent of the contact area and relative speed (amplitude in the case of an oscillator), and proportional to the contact force (normal force). The equation describing the amplitude decay of an harmonic oscillator with a constant friction force is

$A = A_0 - bt$.[3]

The following graph shows that the amplitude decreases linearly with time as does the Q. The Q was numerically calculated from the amplitude by using the most common definition, *i. e.* $Q = 2\pi A^2 / (A_n^2 - A_{n+1}^2)$.



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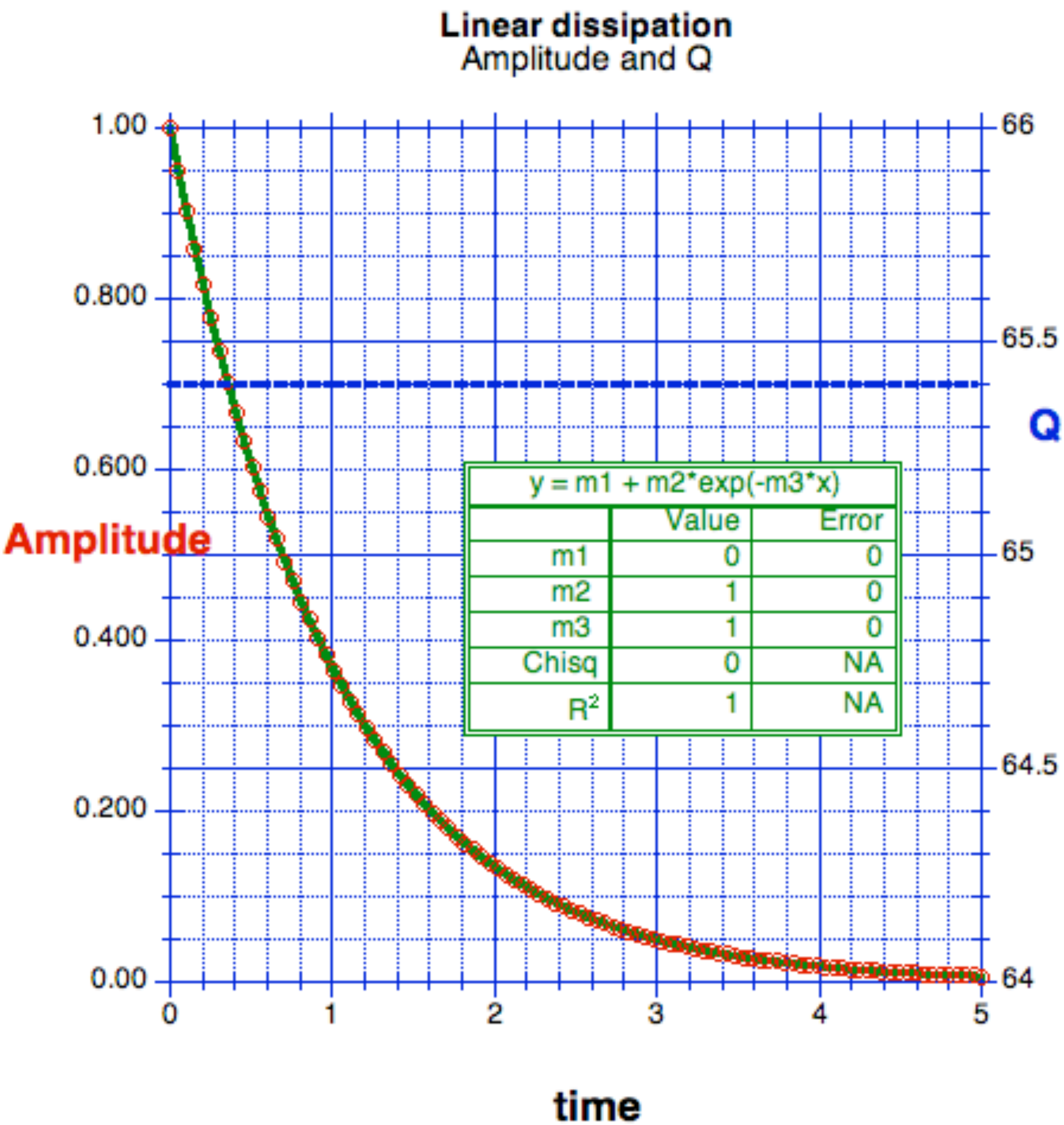
[1] Except mine, of course, HSN 2007-4, p.29

[2] Long before Coulomb *et alii*, he characterized dry friction. Unfortunately, only a few read his note book description.
History of science friction http://www.tribology-abc.com/abc/history.htm

[3] These three amplitude functions are the result of solutions to the differential equation describing the un-driven damped pendulum. A complete description may be found, for example, in Oscillations with three damping effects, Eur. J. Phys. **23** (2002) 155--164.
Oscillations with three damping effects - Abstract - European Journal of Physics - IOPscience

This dissipation type, without exception, begins to dominate near the end of a mechanical oscillator's free decay. If not already obvious, this will become so after understanding the two other principle pendulum dissipations.

The most commonly described pendulum dissipation is Stokes or linear dissipation. It is common because the differential equation's solution is easy[4] and is the correct description of RLC oscillators[5], and magnetic damping. The amplitude follows the familiar exponential decrease: $A = A_0 e^{-bt}$. The next graph shows that exponential decay and the resulting Q.



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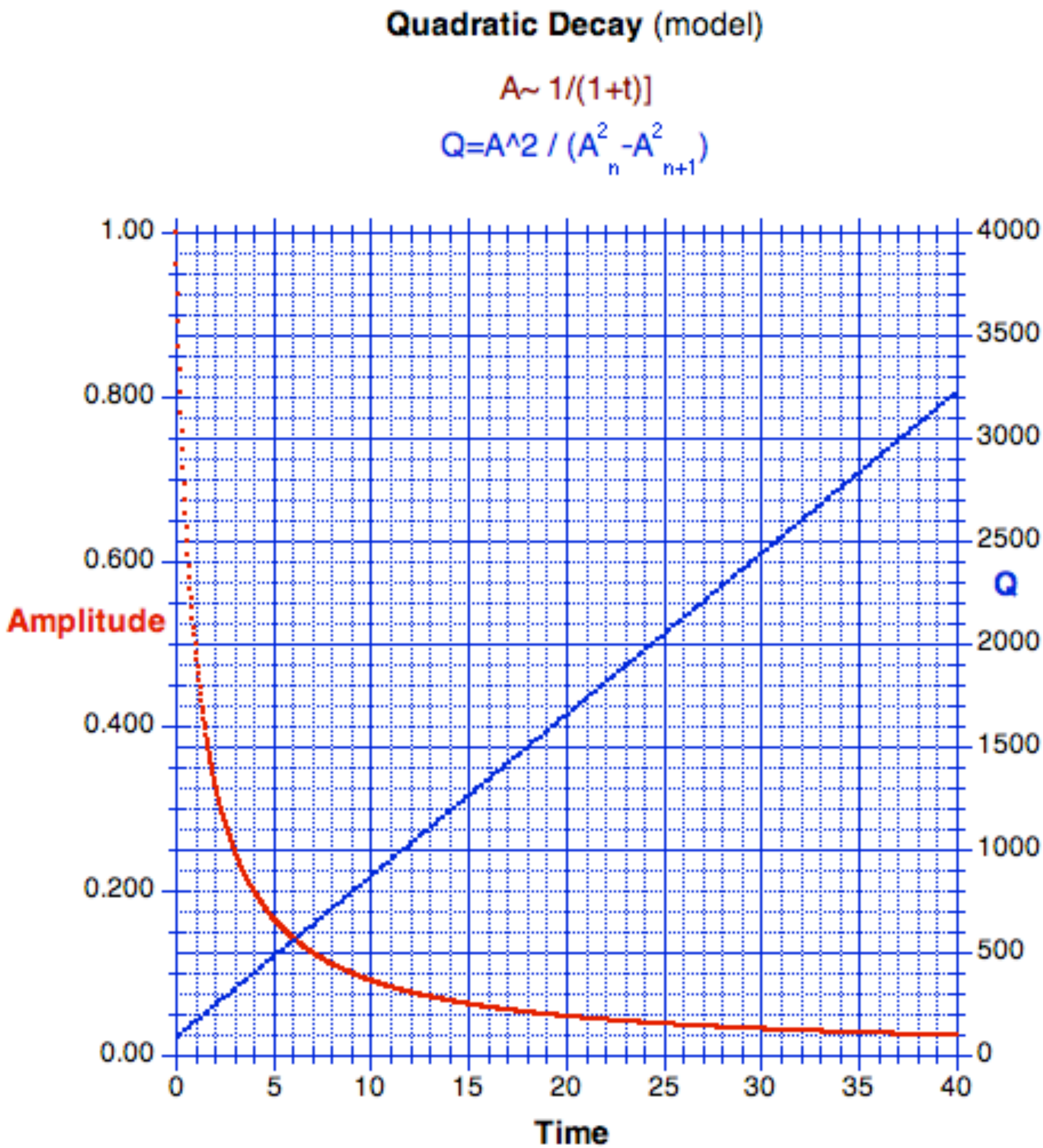
[4] presented in every engineering and physics text I've seen that deals with harmonic oscillators.

[5] Strictly true only for ideal circuit elements, e.g. nearly so, using air core inductors, very low frequencies, etc.

This decay, tho often used in the horological literature does not strictly characterize pendulum decay, as such linear with the speed dissipation only applies to small cross sectional objects moving slowly, i.e. Reynolds numbers approximately less than ten.[6] However, it does, I believe, apply to hysteric dissipation from, for example, the spring suspension, support movement, rod flexure, and, possibly for viscously lubricated pivots-bushings. No lubrication (dry) results in da Vinci dissipation, e.g. dry jeweled bearings.

Finally the quadratic dissipation[7] experienced by large, cross section bobbed and/or amplitude, pendulua -- except those in a vacuum, before the dominant da Vinci dissipation, and often linear dissipation.



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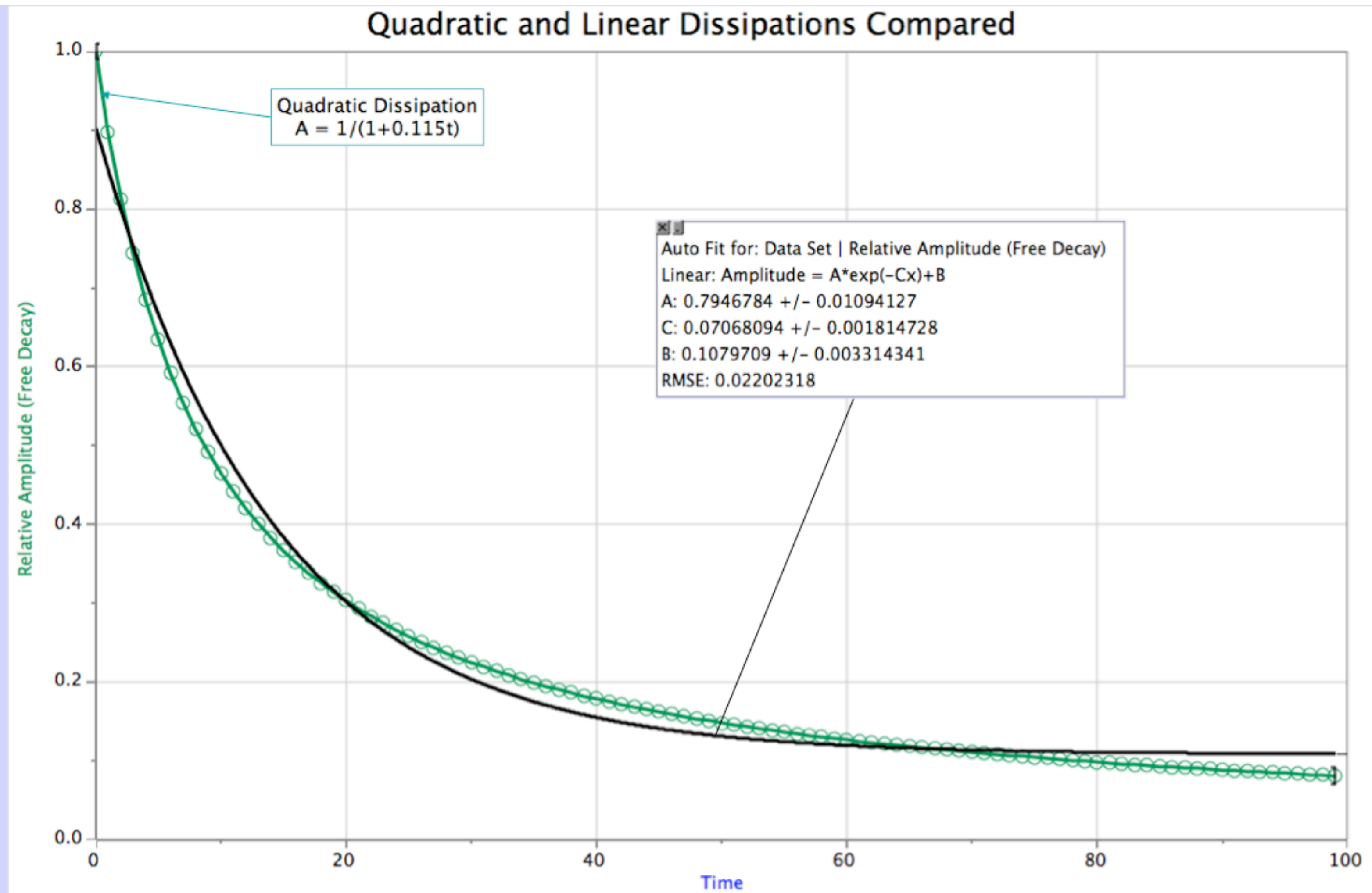
[6] Low Reynolds numbers flows (includes pulsating flows) http://www.hitech-projects.com/euprojects/artic/index/Low%20Reynolds%20number%20flows.pdf

[7] P. Woodward does discuss quadratic dissipation (Non-Linear Air Resistance) in **The Science of Clocks and Watches** (A L Rawlings, third edition) pp. 99, ff.

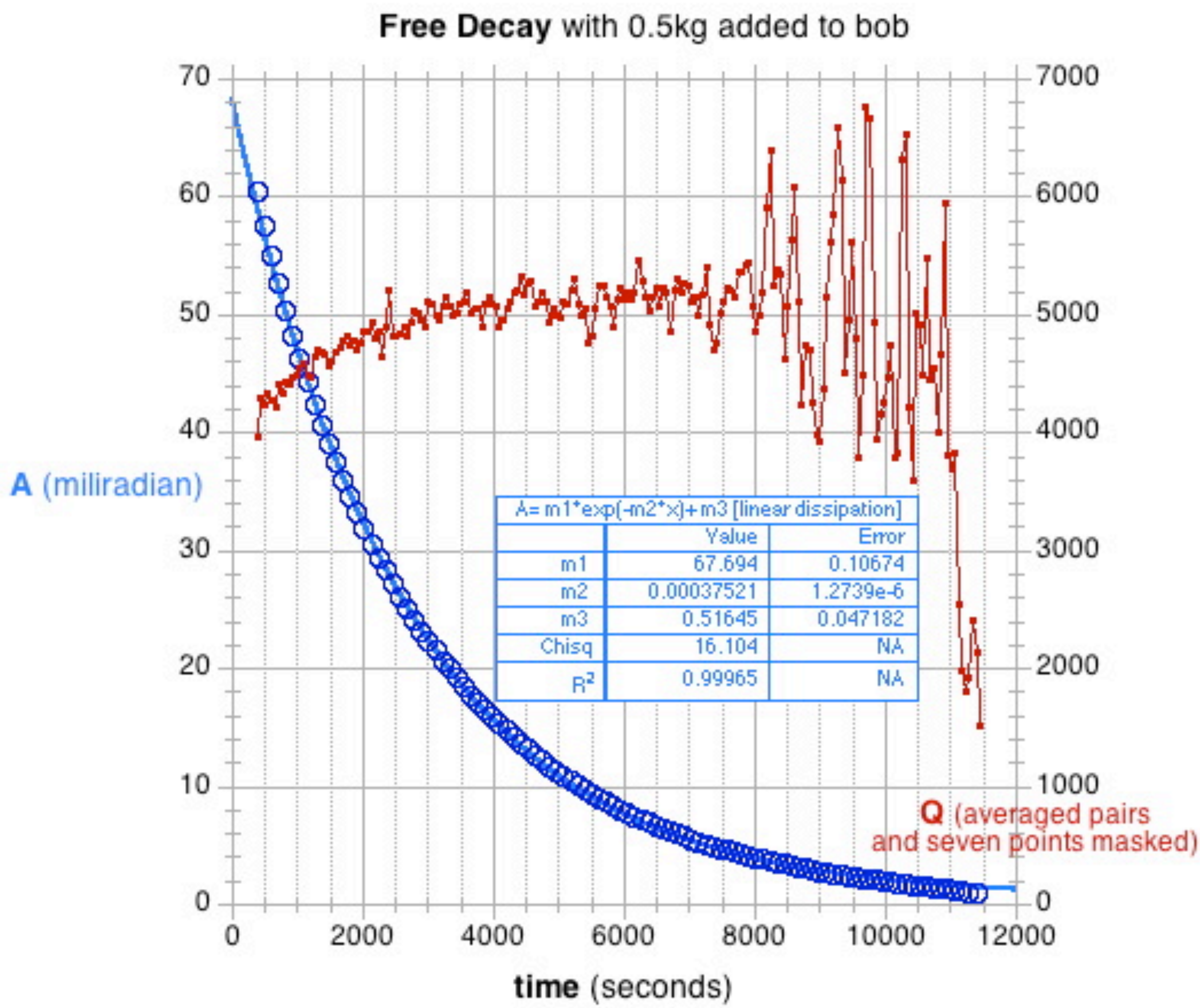
Note the difference: Constant, Linear, and Quadratic, with decreasing amplitude, respectively, Q decreases, is constant, and rises. There are other types of dissipation, which, I think, contribute minimally, and I, therefore, ignore.

Note the similar amplitude behaviour of the linear and quadratic amplitude dissipation models. This may explain their confusion in the literature. The following graph shows this similarity. The quadratic model is fitted by the linear equation. That they are similar is not surprising, as their series expansions are the same to first order. *i.e.* $e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} \cdots$, while $1/(1+x) = 1 - x + x^2 - x^3 \ldots$.



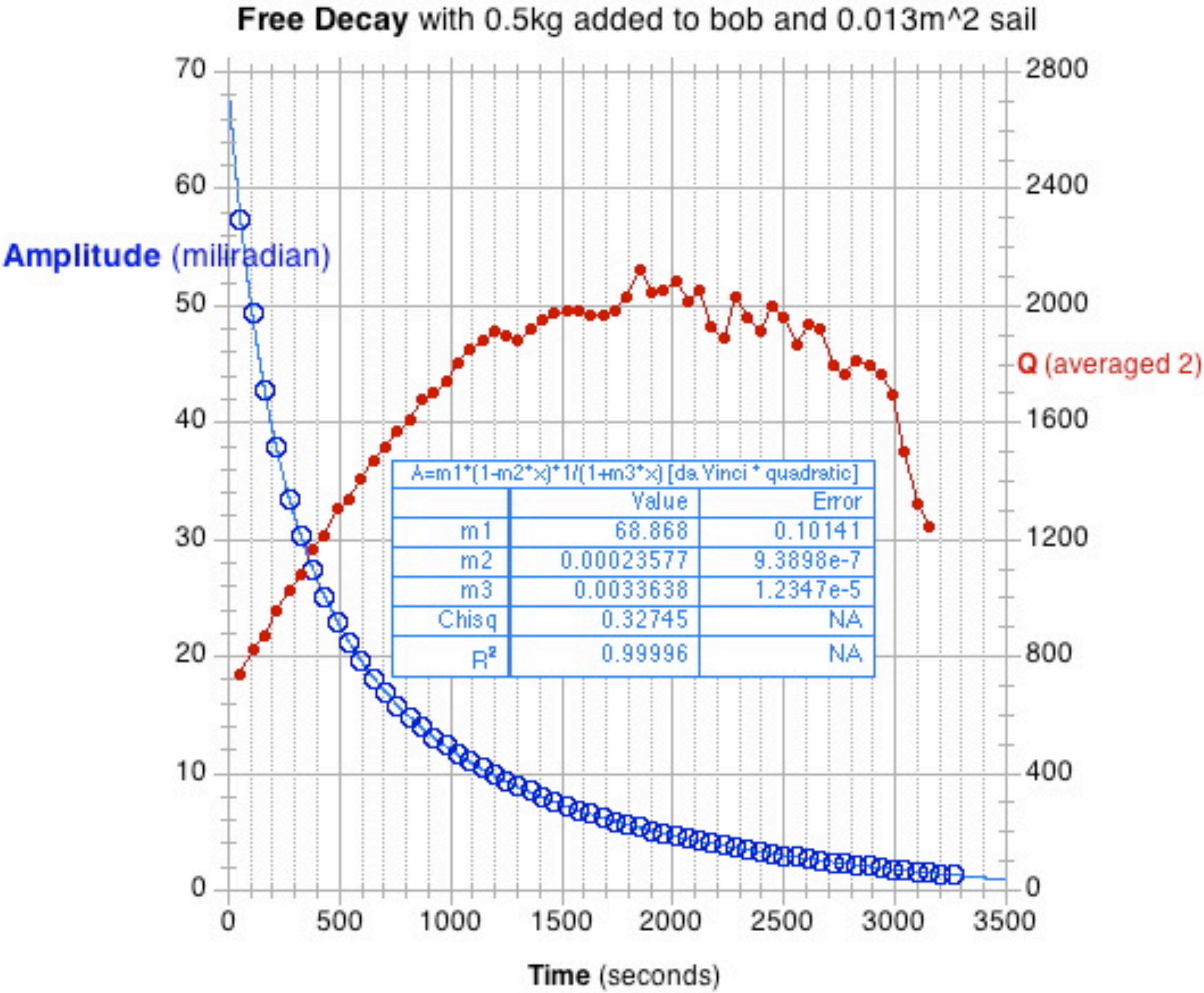
“Armed” with the above, I’ll analyze the decay mechanisms of Gate Keeper’s “Grand Mother” Westminster chimed clock. I’ll begin with the rather contrasting freely decaying Empire pendulum with and with out a large sail.

The graph, below, is of the freely decaying pendulum with an added 0.5kg disk lab. weight. The increased mass overwhelms its added cross sectional area. The data was collected with the MicroSet at time 60. Even so the noise is very great. The data is otherwise not modified except the initial six points and the final one are removed, and adjacent pairs averaged. The first masked because it's difficult to release that pendulum without imparting an out of plane oscillation, and the last is an outlier due to the nature of the calculation of Q.



Since the rising Q is short and weak, the rather good linear fit to the decay is as expected.

Following is the same pendulum, but with an added 0.013 m^2 sail.



Note the very good agreement of the quadratic fit to the decay, as expected from the long and strong initial rising Q.

One can not fail to note the increased quadratic dissipation due to the added sail. It is interesting to note that the sail has very approximately the same cross section as a Synchronome's bob. That the synchronome has about twice the Q (in the linear region) is due to its much more massive bob. Therefore, a conclusion is inescapable.

Here is a photograph of the portion of the pendulum that includes the sail, bob, and the MicroSet's photogate.

And the PhotoGate detail.

This ends part one.