## Q and the "Grand Mother" Clock continued (Part 3)

System Q of weight driven clocks
(As promised in part 2)

This is an exercise (3-45) in Thornton I Marion (5th ed.) p.143.

A grandfather(sic) clock has a pendulum length of 0.7 m and a mass bob of 0.4 kg. A mass of 2 kg falls 0.8 m in seven days to keep the amplitude (from equilibrium) of the pendulum oscillation steady at 0.03 rad. What is the Q of the system? I roughly measured GK's clock and found similar values. The answer reproduced from the solutions manual is[1]:

**3-45.** Energy of a simple pendulum is $\frac{mgl}{2}\theta^2$ where $\theta$ is the amplitude.

For a slightly damped oscillation $\theta(t) \approx \theta\exp(-\beta t)$.

Initial energy of pendulum is $\frac{mgl}{2}\theta^2$.

Energy of pendulum after one period, $T = 2\pi\sqrt{\frac{l}{g}}$, is

$$\frac{mgl}{2}\theta(T)^2 = \frac{mgl}{2}\theta^2\exp(-2\beta T)$$

So energy lost in one period is

$$\frac{mgl}{2}\theta^2\left(1-\exp(-2\beta T)\right) \approx \frac{mgl}{2}\theta^2 2\beta T = mgl\theta^2\beta T$$

So energy lost after 7 days is

$$mgl\theta^2\beta T\frac{(7\text{ days})}{T} = mgl\theta^2\beta(7\text{ days})$$

This energy must be compensated by potential energy of the mass $M$ as it falls h meters:

$$Mgh = mgl\theta^2\beta(7\text{ days}) \quad\Rightarrow\quad \beta = \frac{Mh}{ml\theta^2(7\text{ days})} = 0.01\text{ s}^{-1}$$

Knowing $\beta$ we can easily find the coefficient $Q$ (see Equation (3.64))

$$Q = \frac{\omega_R}{2\beta} = \frac{\sqrt{\omega_0^2 - 2\beta^2}}{2\beta} = \frac{\sqrt{\frac{g}{l} - 2\beta^2}}{2\beta} = 178$$

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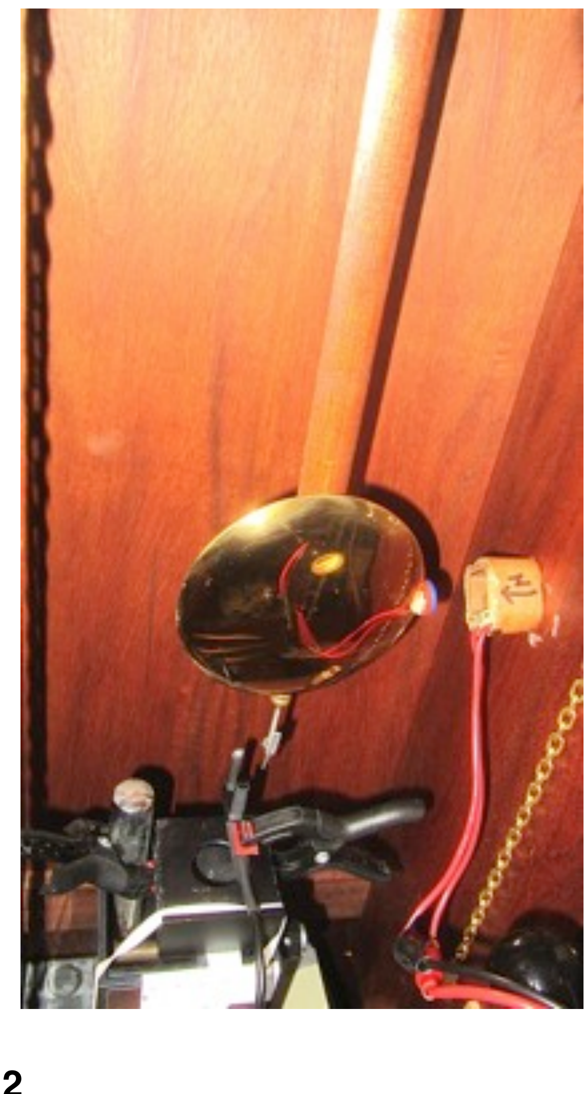
[1] Note the assumption of linear dissipation. Intuitively, I expect little error tho these pendulua may have significant quadratic dissipation.

I had hoped to measure the Q, or loss, of a suspension spring, as I wrote in part 2, but found it not practical.  This was because the spring will not support the necessary mass for a low frequency.  At high frequencies the dissipation is largely due to air drag instead of loss in the spring.  To measure this requires a considerably more sophisticated system.  However, I have something much more interesting, I pray.

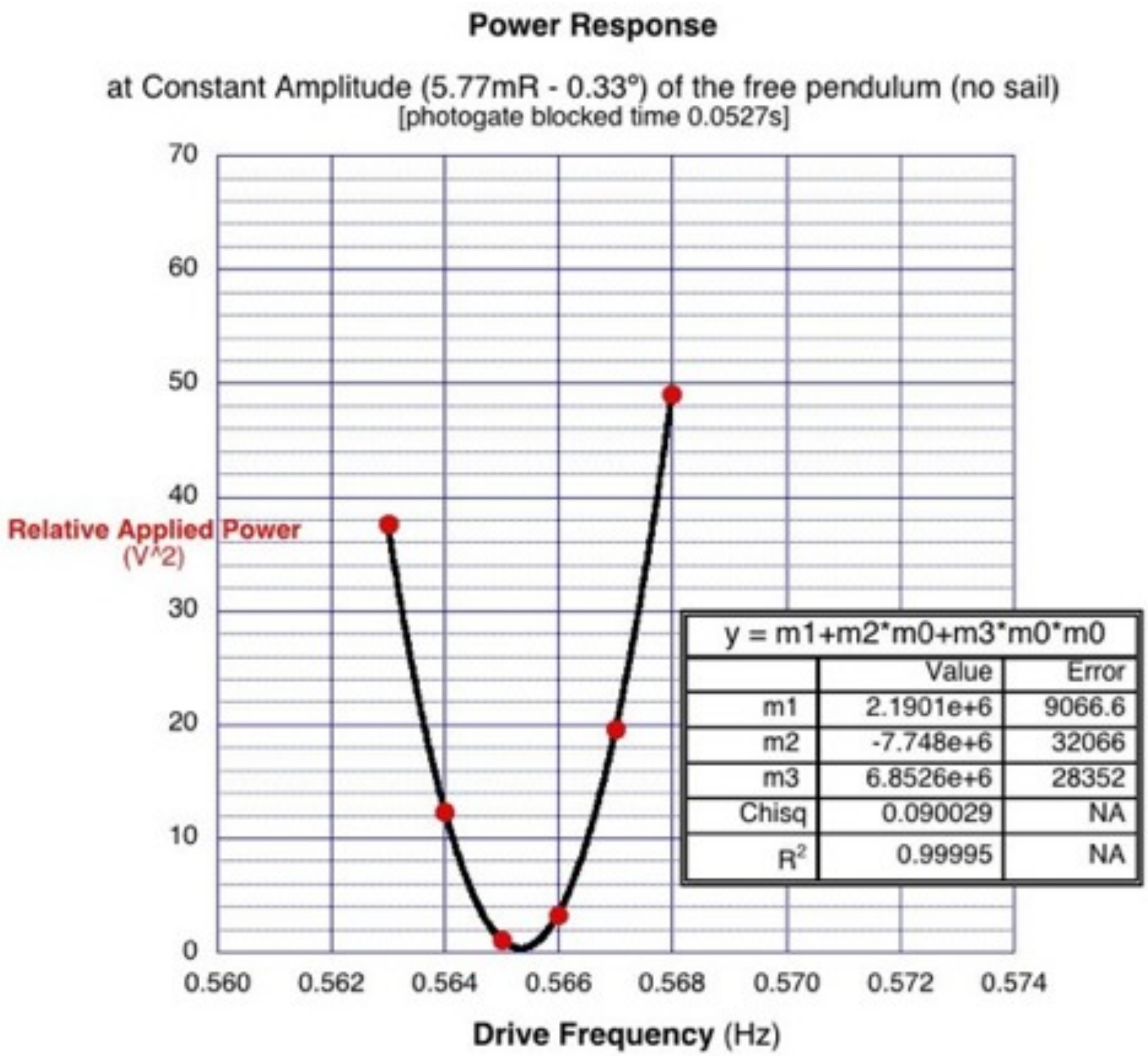
Measuring the GM clock's Q using the band width method.

Several HSN horologists "loudly" proclaim pendula have no single resonance frequency. So what else is new!  Of course, that's the nature of nonlinear oscillators.  However, one may find the bandwidth method Q simply by maintaining a constant amplitude during the measurements. Not only does this remove circular error, but more importantly the amplitude variation of Q when, as usual, clock pendulua's dissipation is not linear.  I found three Q values by manually adjusting the power and frequency of the electro-magnetic drive.  The pendulum was free for the three (crutch, etc. removed), two with an added sail to reduce the Q at two rather different amplitudes and the third without the sail and it's amplitude the same as the smaller amplitude sail's trial.

Here are images showing the pendulum, drive and photogate:

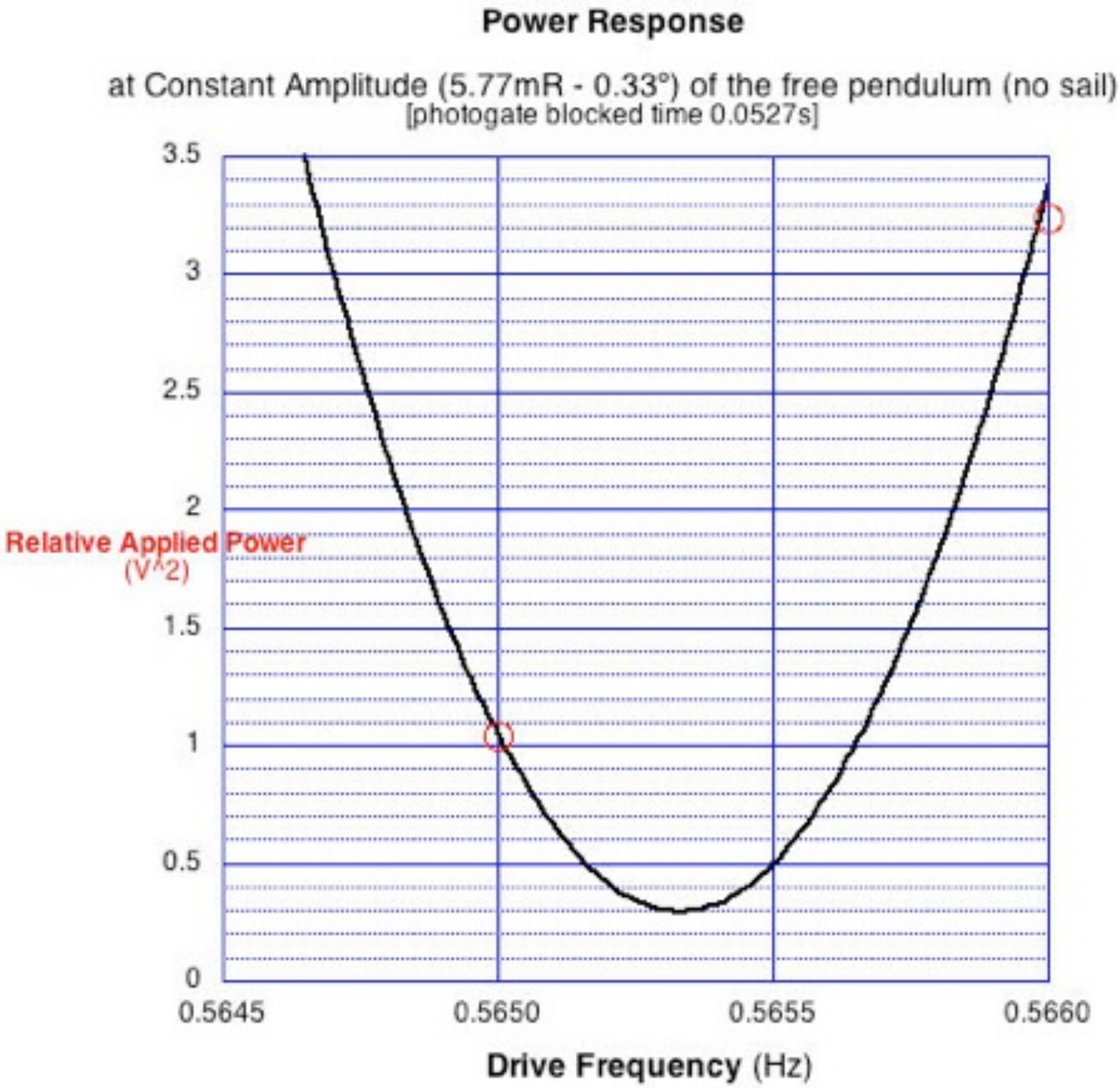


I found the bandwidth (FWHP) by finding the frequencies, using a quadratic fit, for double the EMF required to maintain the same amplitude as that at resonance. The following graphs are of the three trials:




| | Value | Error |
|---|---|---|
| m1 | 2.1901e+6 | 9066.6 |
| m2 | -7.748e+6 | 32066 |
| m3 | 6.8526e+6 | 28352 |
| Chisq | 0.090029 | NA |
| $R^2$ | 0.99995 | NA |

The graph below is of the one above with an expanded scale.



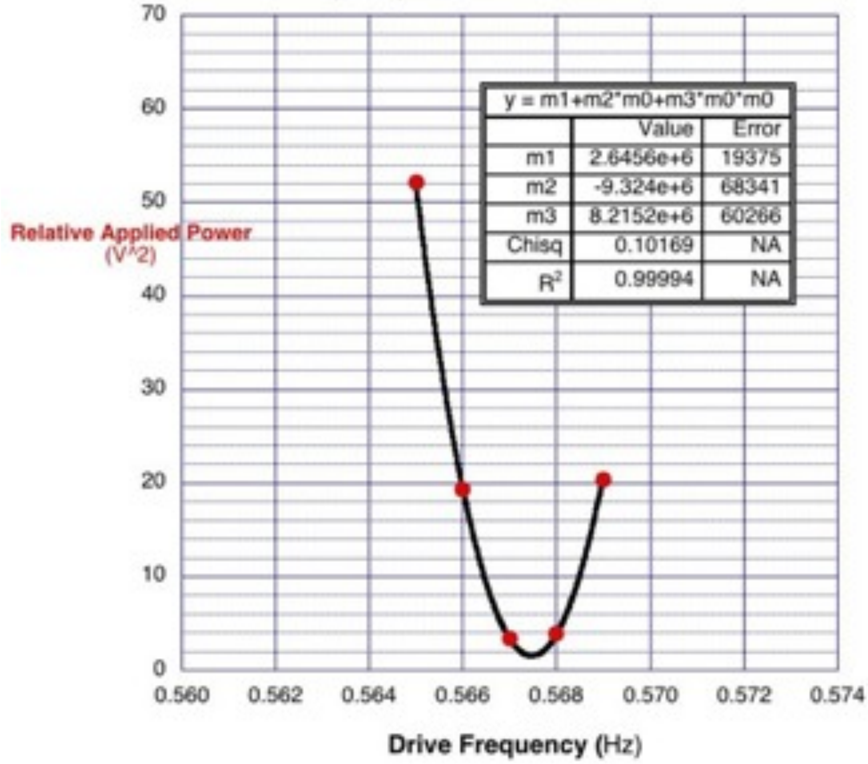

The next graph is of the trial with the same amplitude as above, but with an added sail. Below it is the expanded graph.

The sail is as pictured above.

And the last two are of the trial with the sail and at a somewhat greater amplitude.

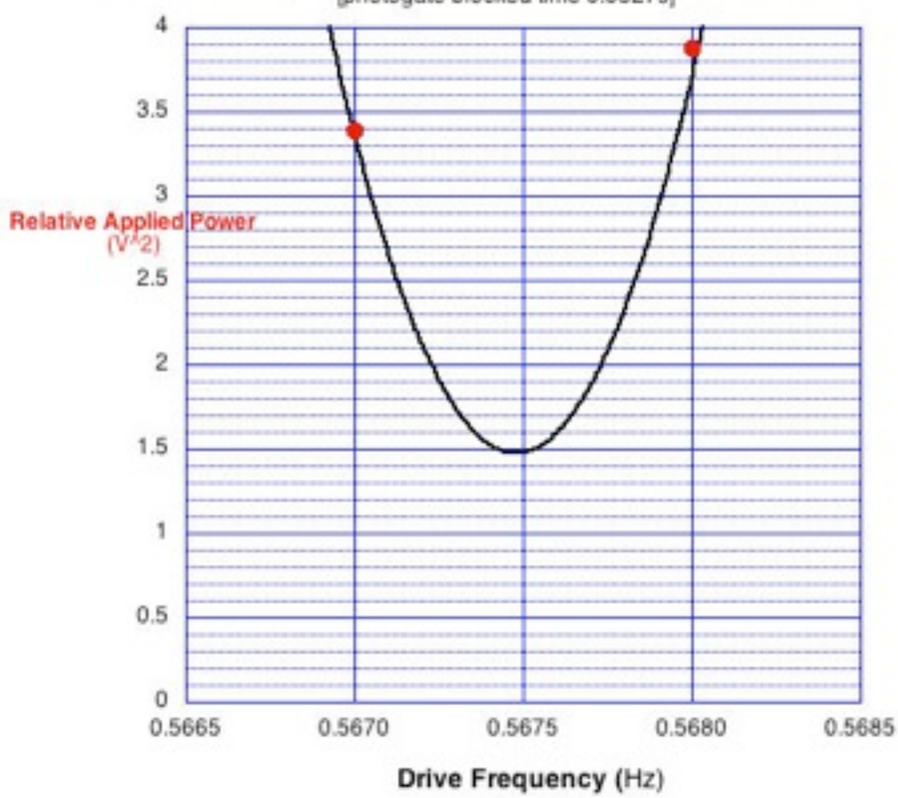
**Power Response**

at Constant Amplitude (5.77mR - 0.33°) of the free pendulum with sail
[photogate blocked time 0.0527s]



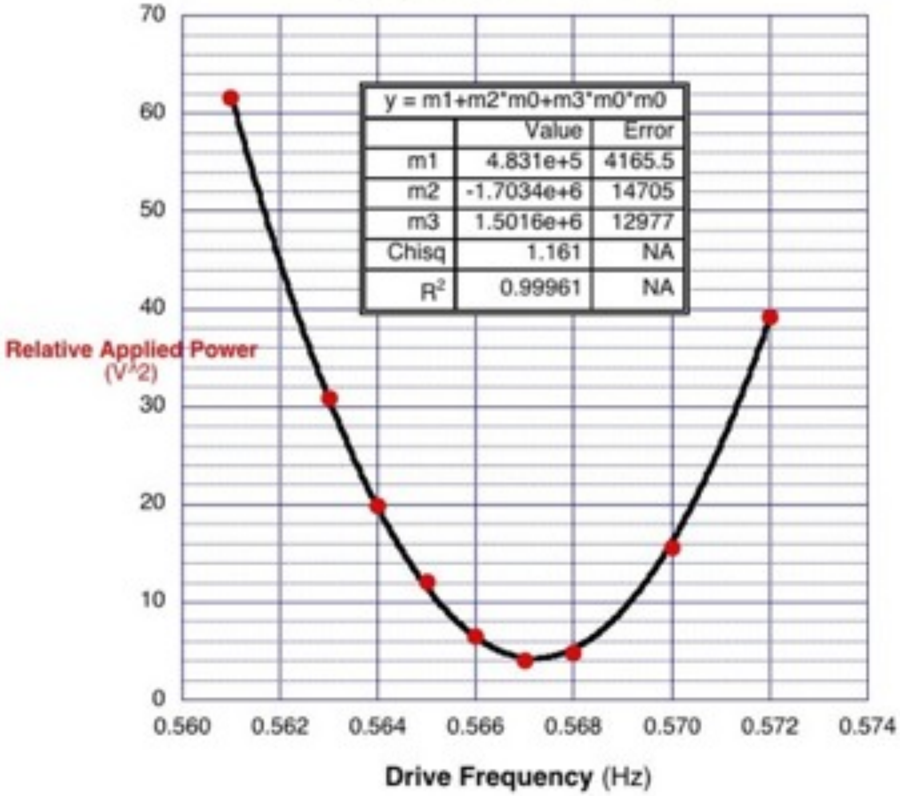

**Power Response**

at Constant Amplitude (5.77mR - 0.33°) of the free pendulum with sail
[photogate blocked time 0.0527s]



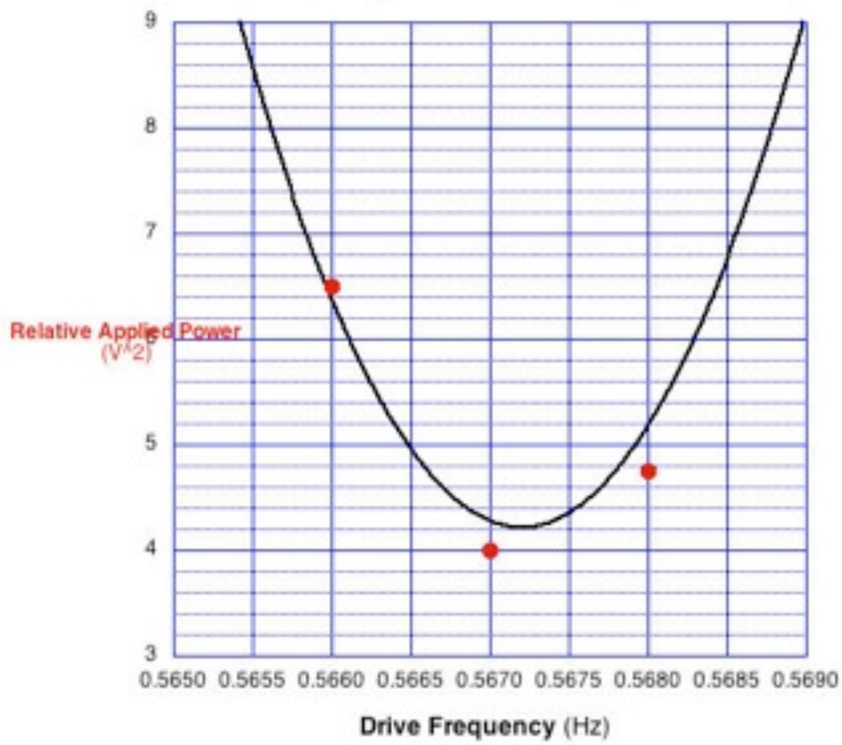
**Power Response**

at Constant Amplitude (~21mR - 1.2°) of the free pendulum with sail
[photogate blocked time 0.0147s]




**Power Response**

at Constant Amplitude (~21mR - 1.2°) of the free pendulum with sail
[photogate blocked time 0.0147s]



The respective Qs calculated from the fitted values are: 1350, 709, and 189. They, unfortunately, don't agree well with the Qs found by free decay at the given amplitudes by the "bc" method. Which are, respectively, 1540, 620, and 220. I'm rather certain the "bc" method is considerably more accurate, as the resolution of the MicroSet -- photogate system is approximately a microsecond, and is the average of very many measurements, while the bandwidth method has poor resolution (one mHz and 10 mV), depends on a small difference of two large numbers and only a few data points, tho the fits are remarkable good.

bc

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