Using 3-cm microwaves for optics laboratory experiments

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Commercially available 3-cm microwave equipment can produce predictable diffraction patterns if precautions are taken to eliminate standing waves resulting from reflections from various surfaces. Equipment modifications and operating procedures are described for a student laboratory experiment involving single-slit diffraction in the transition range between Fraunhofer and Fresnel patterns. Polarization measurements are also described.

I. INTRODUCTION

Equipment is commercially available,[1] which makes it feasible for students to study wave phenomena using electromagnetic waves of approximately 3-cm wavelength. With such equipment physical optics laboratory exercises can be performed using components that are more conveniently modified and adjusted than those needed for working with visible light. However, experience in one of our intermediate-level laboratory courses has shown that the commercially available equipment does not permit one to obtain results which correspond to the predictions of conventional optical theory.

We wished the students to be able to study single-slit diffraction patterns produced by a range of slit widths; from slits, where the diffraction patterns approach the Fraunhofer limit, to wider slits, where the pattern is described by the Fresnel diffraction equations. Texts[2] usually treat Fraunhofer and Fresnel diffraction in separate sections, but we wished to have our students observe that there is no sharp boundary between the two.

From our initial observations of single-slit diffraction patterns we saw that the wave intensity at the receiver as a function of diffraction angle did not agree with expectations. Curve (a) in Fig. 1 shows the receiver output as a function of diffraction angle using the original slit assembly with a slit width of 6.0 cm. A comparison with curve (c), calculated from Fresnel theory, shows that the observations differ in several respects from the calculated curve. From a series of tests it was found that the principal difficulty was unwanted reflections of waves from various surfaces in the system. A similar conclusion was reached when investigating the polarization of the 3-cm waves.

II. REDUCTION OF REFLECTIONS AND STANDING WAVES

Because the horn on the receiver has an entrance that is 9.2 cm wide, the receiver meter readings are influenced by the wave intensities extending over diffraction angles of several degrees when the receiver is a few tens of centimeters from the diffracting slit. This obscures some of the variations in intensity as a function of diffraction angle produced by slits that are several wavelengths wide. To enable these details to be observed, a metal plate having a 2 cm wide opening was placed on the front of the receiver horn. This opening extends across the narrow dimension of the horn which is normally vertical. With this reduction in the width of the horn entrance, data recorded at 2° intervals, with the horn at approximately 30 cm from the slit, resulted in no loss in pattern detail.

On the contrary, observations under these conditions resulted in more variations in intensity than predicted, as shown by curve (b) in Fig. 1. Also, the intensity observed near the centerline (0°) was a strong function of the slit-to-horn distance. This indicated the presence of standing waves between the slit plates and the plate on the entrance of the receiver horn. This situation does not normally occur with equipment using visible light because slit edges and other components have small surface irregularities, which prevent standing wave patterns at visible wavelengths but are negligible for 3-cm wavelengths. Because the reflections could not be eliminated, the only way the standing waves could be avoided was to direct the waves reflected by the receiver so that they did not strike the slit plates. A reflecting plate 13-cm square, placed immediately in front of the horn and tilted at 45° to the vertical, reduced the undesired effects to a negligible level. Because of diffraction effects, it is necessary that this reflecting plate have length and width that are several wavelengths so that the reflected waves travel in the desired direction. Of course, this plate in front of the receiver horn needed a slot in it slightly wider than the 2-cm opening in the horn cover plate, mentioned above.

Another source of reflections which resulted in standing waves contributing to unpredictable intensities was the

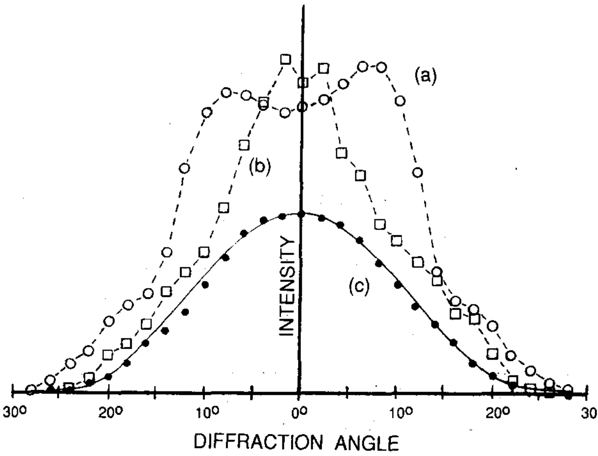

Fig. 1. Wave intensity as a function of diffraction angle for a narrow slit ($\Delta V = 1.13$) under various conditions: (a) Open circles, using original slit, unmodified receiver horn and supports. (b) Squares, with 2-cm wide aperture on receiver horn, also new slit and slit supports. (c) Solid curve calculated from Fresnel equations. Solid dots are observations taken using the changes in equipment described in the text. Vertical scales are arbitrary. Dotted curves serve to connect data points. Here, and in Fig. 4, the heights of the theoretical curves were chosen to match the experimental curves at the centers of the patterns.

support stand for the receiver. To redirect waves striking this stand, the front of the stand was covered with a V-shaped piece of metal having the vertex of the V toward the incident waves. A 60° angle between the arms of the V produced satisfactory results.

The person operating the equipment can also reflect the waves. To avoid noisy data from operator motion, the operator should stand a few feet from the equipment, preferably in the plane of the slit plates so as to be in a region of low field intensity. In this position it is convenient to read the receiver output signal on a 3-V dc multimeter, rather than using the meter built into the receiver. The equipment should be located well away from the walls of the room and the axis of the equipment should be oriented at 45°, with respect to the walls.

III. OTHER EQUIPMENT MODIFICATIONS

The slit plates in our original equipment were thin sheets of steel which were held in the desired locations by attraction to plastic strip magnets incorporated in a supporting frame. We found it to be rather difficult to adjust these plates so that the slit was the desired width, the slit was centered at the center of the angle scale used for measuring the receiver location, and the slit edges were parallel. In some cases, a millimeter error in position would produce an appreciable effect on the diffraction pattern. These problems were reduced by replacing the magnetic plate holder by one using mechanical guides and clamps and incorporating a millimeter scale to measure slit width and centering.

The small original angle scale did not permit adequate receiver position measurement so it was replaced by a scale drawn on the table top with a radius equal to the length of the arm to which the receiver assembly was clamped.

The dots near curve (c) in Fig. 1 show the results of observations made with all the modifications of the equipment in use.

One of the usual assumptions made in developing the equations describing diffraction by a slit is that the incident radiation has uniform intensity across the width of the slit. The original equipment did not satisfy this condition. Measurements showed that, as expected, the radiation from the source horn was concentrated in the forward direction, as illustrated in Fig. 2. It was found that the radiation intensity at the plane of the slit could be made constant to within 6% over a 20-cm wide slit by placing a pair of thin metal vanes in front of the source horn, as shown in Fig. 3. However, without using these vanes, slits up to 13 cm wide produced diffraction patterns that agreed well with the Fresnel theory, as shown in Fig. 4. For slit widths of 16 and 20 cm, the vanes improved the agreement of the diffraction pattern with predictions, but peak positions were displaced to larger angles and the peak amplitudes differed from those predicted. This effect was noticeable with the 13-cm slit and was much larger with the 16- and 20-cm slits.

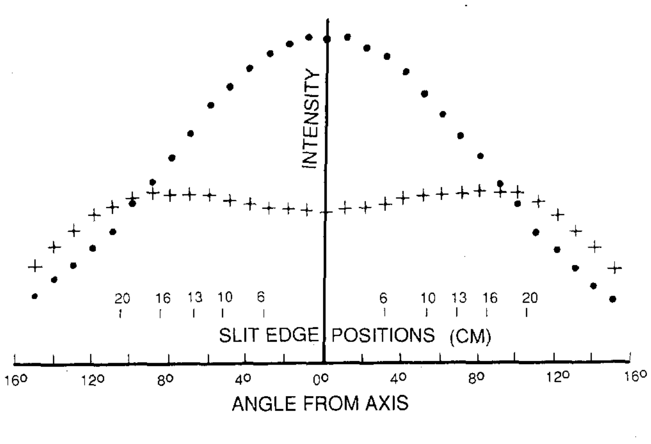

Fig. 2. Intensity of waves emitted from source horn as a function of the angle from the axis of the source horn. Solid dots are for the unmodified horn. Plus dots illustrate the effect of the vanes shown in Fig. 3. The scale of numbers from 6 to 20 indicates the edge positions of slits having various widths in centimeters. Intensity scale is arbitrary.

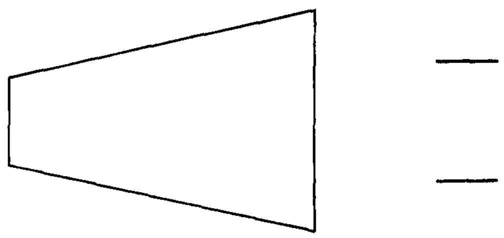

Fig. 3. Top view of source horn and two parallel vertical vanes. Vane widths are 2.0 cm, separation is 4.0 cm, and the separation from the wide end of the horn is 4.0 cm. Vane vertical length is 17 cm.

IV. CALCULATED DIFFRACTION PATTERNS

To facilitate comparison between the observed diffraction patterns and the predictions of Fresnel theory, a computer program SLIT was written in BASIC. The Fresnel integrals were approximated using the expressions 7.3.9 and 7.3.10 with the approximations 7.3.32 and 7.3.33 found in Ref. 3.

The distance from the plane of the slit to the receiver was well defined by the plate having the 2-cm wide aperture on the receiver horn. However, the distance from the plane of the slit to the source of waves was not expected to be determined by the distance to the wide end of the source horn because that is not a point source of waves. For relatively narrow slits the shape of the diffraction pattern is not a rapid function of this distance, but for wider slits (for example, 16 cm wide) the relative heights of the peaks in the pattern is a sensitive function of source distance. A comparison of the calculated diffraction pattern with observations resulted in the most satisfactory agreement if the distance from slit to source was measured to the narrow end of the horn. This distance was used in calculating all the curves shown in Fig. 4.

For each diffraction pattern in Fig. 4 the slit width is given in centimeters and also in the dimensionless parameter $\Delta V = W[2(A+B)/\lambda AB]^{1/2}$, which is commonly used[2] to designate relative slit widths. Here, W is the slit width, λ is the wavelength, A the distance from source to slit, and B the distance from slit to receiver.

As can be seen from Fig. 4, for $W = 6.0$ cm ($\Delta V = 1.16$) the Fresnel and Fraunhofer calculations are practically indistinguishable over the range studied and the observations fit the curve well. For $W = 10.0$ cm ($\Delta V = 1.93$) the Fraunhofer calculation shows the point of zero intensity at 16.6°, while the Fresnel calculation and the observations have no zero.

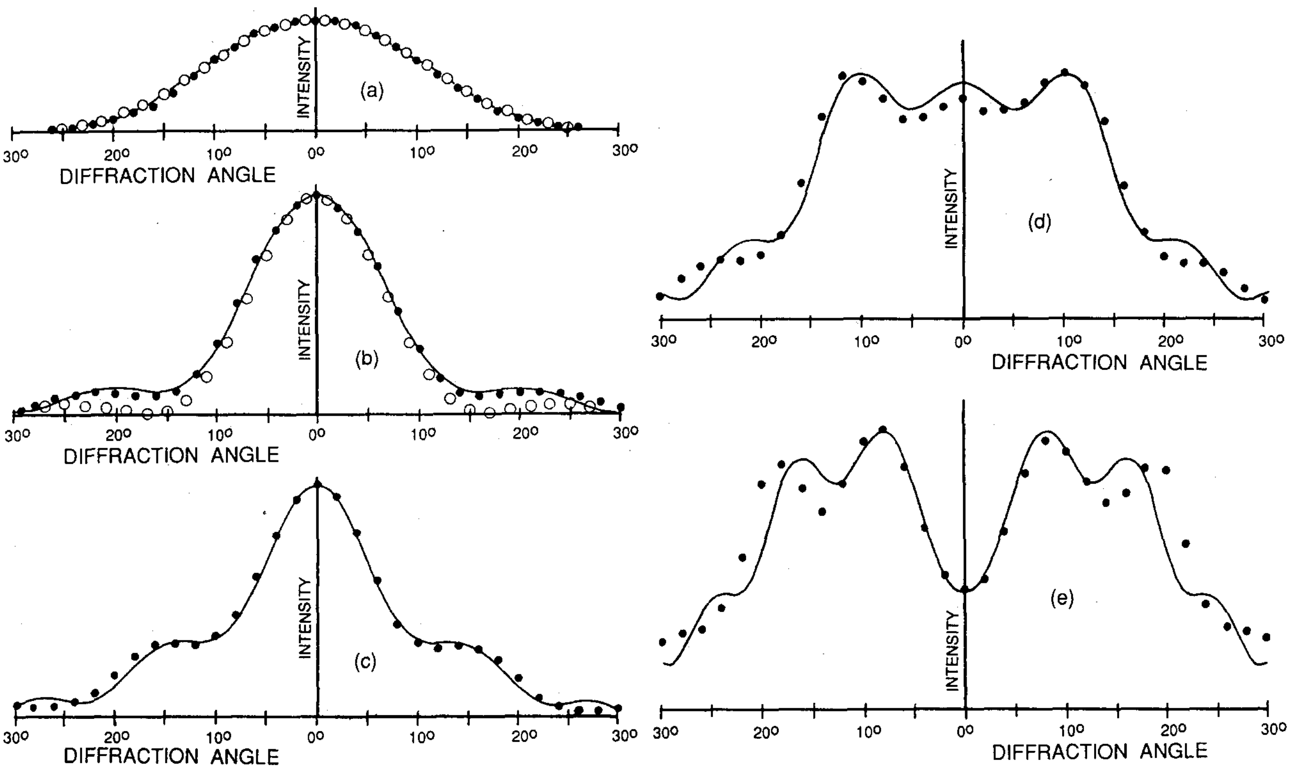

Fig. 4. Wave intensity as a function of diffraction angle for several slit widths. Solid dots are observations. Solid curves were calculated using Fresnel equations. Open circles were calculated using the Fraunhofer equation. (a) slit width = 6.0 cm, $\Delta V = 1.16$; (b) slit width = 10.0 cm, $\Delta V = 1.93$; (c) slit width = 13.0 cm, $\Delta V = 2.51$; (d) slit width = 16.0 cm, $\Delta V = 3.09$; (e) slit width = 20.0 cm, $\Delta V = 3.86$. Intensity scales are arbitrary.

V. POLARIZATION MEASUREMENTS

The electromagnetic waves emerging from the source horn are polarized with the electric vector parallel to the short dimension of the horn opening, and the receiver responds to the component of the incident waves, which is in that direction. From these considerations, one would expect that the output voltage of the receiver would be proportional to $\cos^2 \phi$, where ϕ is the angle between the directions of the short dimensions of the source and receiver horns. The receiver sensitivity calibration curves, Fig. 5, were obtained making use of this expected relationship. The intensity scale contains an arbitrary constant.

The original equipment provided a polarization analyzer plate having slots 0.6 cm wide separated by 0.6 cm. No provision was made for using this plate with values of ϕ other than 0°, 45°, and 90°. To make it possible for the students to investigate in more detail the angular dependence of the wave transmission through a polarization analyzer, a wire grid was made using wires 0.08-cm diameter with center-to-center spacing of 0.40 cm. These wires were stretched across a ring having a diameter of 15 cm. To reduce standing waves produced by reflections from the ring and its support, these were made as thin as feasible consistent with the necessary rigidity.

It was also found that the small angle reflection of waves from the table top had a strong influence on the receiver output. To reduce this effect, new support stands for the source and receiver were made, which increased the distance of the source and receiver horns above the table top from 18 to 30.5 cm.

In spite of these precautions, we noticed that the receiver

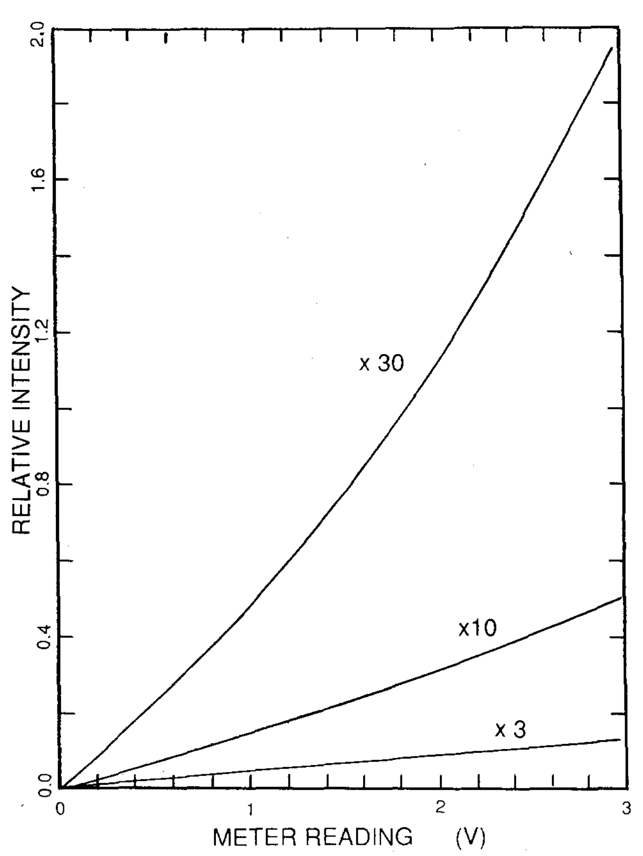

Fig. 5. Receiver calibration curves; wave intensity versus receiver output voltage. The numbers on each curve are those on the sensitivity control knob. The intensity scale on the graph has an arbitrary factor.

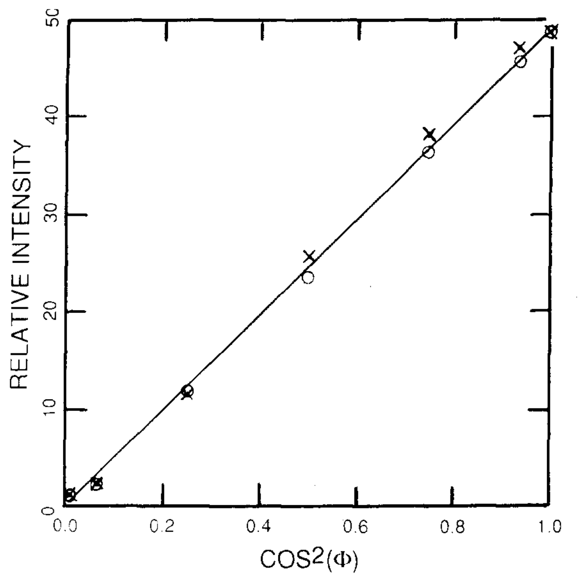

Fig. 6. Transmission of waves through the wire grid polarization analyzer. ϕ is the angle between the grid wires and the horizontal when the E vector of the waves is vertical. The open circles are for clockwise rotation of the grid viewed from behind the receiver; the X dots are for counterclockwise rotation.

output voltage when using the analyzer grid was a function of the distance of the grid from the receiver. To obtain consistent results, it was found necessary to make small adjustments in the grid-to-horn distance at each value of ϕ so as to maximize the receiver output.

With these modifications and procedures the wave intensity at the receiver followed the $\cos^2 \phi$ relationship for analyzer grid rotation, as shown in Fig. 6.

VI. CONCLUSIONS

Our experience in using 3-cm electromagnetic waves shows the necessity of reducing wave reflections from equipment components and surroundings in order to obtain results consistent with theoretical predictions. With the equipment modifications and operating procedures described, students can obtain single slit diffraction patterns for a range of slit widths that cover the transition from Fraunhofer to Fresnel diffraction and also investigate the polarization of the waves.

A listing of the program SLIT may be obtained from the author.

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[1] The equipment on which this is based was obtained from Pasco Scientific, 10101 Foothills Blvd., Roseville, CA 95661. Similar equipment may be obtained from Sargent-Welch Scientific Co., 7300 N. Linder Ave., Skokie, IL 60077; from Central Scientific Co., 11222 Melrose Ave., Franklin Park, IL 60131; and other suppliers of educational equipment.

[2] For example: E. Hecht, *Optics* (Addison-Wesley, Reading, MA, 1987), 2nd ed.; G. R. Fowles, *Introduction to Modern Optics* (Holt, Rinehart and Winston, New York, 1975), 2nd ed.; C. L. Andrews, *Optics of the Electromagnetic Spectrum* (Prentice-Hall, Englewood Cliffs, NJ, 1960).

[3] *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1965), pp. 301–302.

Least-squares fitting when both variables contain errors: Pitfalls and possibilities

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Least-squares fitting is reviewed, in tutorial form, when both variables contain significant errors. Various error models are described; corresponding appropriate weighting is discussed; and the interpretation of weighting is clarified by a physically intuitive description and by graphical results. Resources in the literature on least-squares fitting that are suitable for physics and astronomy students are reviewed. Algorithms for straight-line fitting, indicate practical solution methods, are summarized and numerical comparisons are given. Also described are several readily available computer programs that allow fitting for both straight-line and nonlinear situations and that are appropriate for both research and teaching applications.

I. INTRODUCTION

Least-squares fitting when both variables have errors is a perennially interesting problem, on which a dozen communications have been published in this Journal in the past 2 decades.[1-12] There is also an extensive research literature on the subject, dating back more than a century. The problem is often called generalized least squares in the physical sciences and the errors-in-variables (EOV) model in statistics.[13] Such least-squares fitting is also of significant interest in astronomy[14] and in chemistry.[15] For students in the physical sciences it is important to learn the pitfalls and possibilities in least-squares analyses, especially since many of the algorithms are now available in user-friendly