Metallic Delay Lenses

By WINSTON E. KOCK

A metallic lens antenna is described in which the focussing action is obtained by a reduction of the phase velocity of radio waves passing through the lens rather than by increasing it as in the original metal plate lens. The lens shape accordingly corresponds to that of a glass optical lens, being thick at the center and thin at the edges. The reduced velocity or "delay" is caused by the presence of conducting elements whose length in the direction of the electric vector of the impressed field is small compared to the wavelength; these act as small dipoles similar to the molecular dipoles set up in non-polar dielectrics by an impressed field. The lens possesses the relatively broad band characteristics of a solid dielectric lens, and since the conducting element can be made quite light, the weight advantage of the metal lens is retained. Various types of lenses are described and a theoretical discussion of the expected dielectric constants is given. An antenna design which is especially suitable for microwave repeater application is described in some detail.

Introduction

THE metal lens antennas described by the writer elsewhere[1] comprised rows of conducting plates which acted as wave guides; a focussing effect was achieved by virtue of the higher phase velocity of electromagnetic waves passing between the plates. Higher phase velocity connotes an effective index of refraction less than unity, and a converging lens therefore assumes a concave shape. The relation between the index of refraction n, the plate spacing, a, and the wavelength λ

$$n = \sqrt{1 - (\lambda/2a)^2}, \tag{1}$$

indicates that the refractive index varies with wave length. As a consequence, such lenses exhibit "chromatic abberation"; i.e. the band of frequencies over which they will satisfactorily operate is limited. Although some of these lenses may have ample bandwidth (15% to 20%) for most microwave applications, others, having large apertures in wavelengths, may have objectionable bandwidth limitations. For example, the lens of Fig. 1, having an aperture diameter of 96 wavelengths, has a useful bandwidth of only 5%.

As a means for overcoming these band limitations, the metallic lenses of this paper were developed. They are light in weight and possess an index of refraction which can be made sensibly constant over any desired band of microwave frequencies. They avoid the weight disadvantages of glass or plastic lenses, and retain the tolerance and shielding advantages of the lens

[1] W. E. Kock, *Bell Laboratories Record*, May 1946, p. 193; *Proc. I. R. E.*, Nov. 1946, p. 828.

over the reflector antenna. Because electromagnetic waves passing through them are slowed down or "delayed" (as in the glass lenses of optics), they are called delay lenses; and since the elements in the lenses which produce the delay are purely metallic, they are called metallic delay lenses.

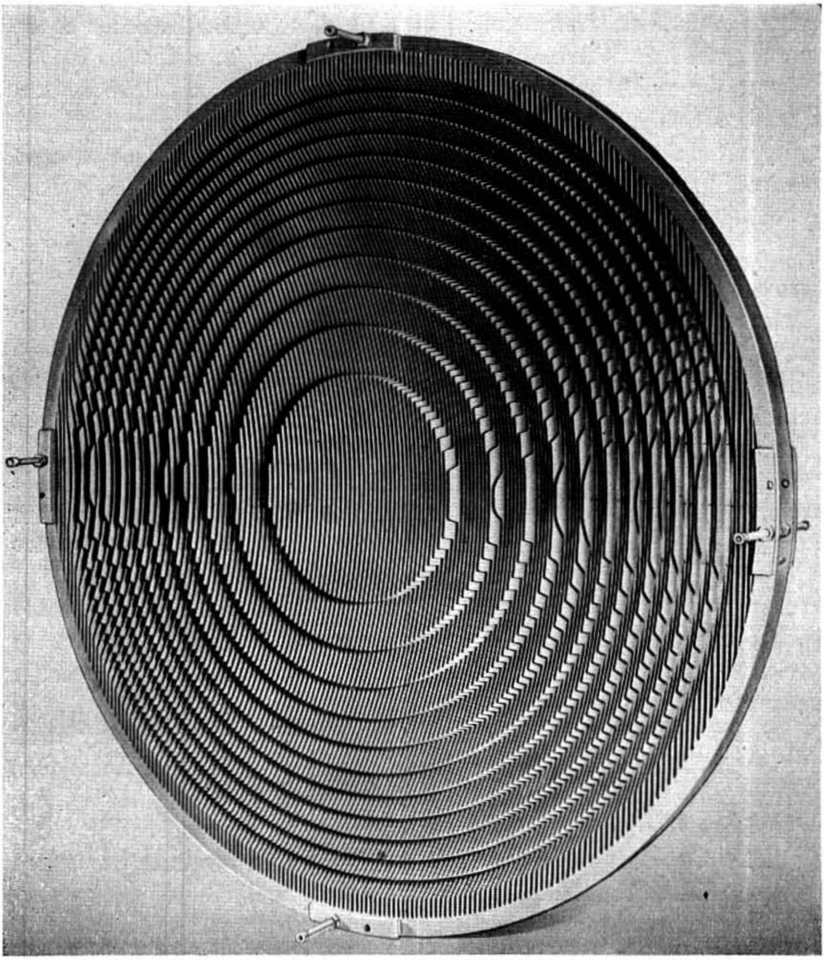

Fig. 1—Waveguide metallic lens having an aperture of 96 wavelengths and a useful bandwidth of 5%.

PART I—EXPERIMENTAL

Fundamental Principles

The artificial dielectric material which constitutes the delay lens was arrived at by reproducing, on a much larger scale, those processes occurring in the molecules of a true dielectric which produce the observed delay of electromagnetic waves in such dielectrics. This involved arranging metallic elements in a three-dimensional array or lattice structure to simulate the crystalline lattices of the dielectric material. Such an array responds to radio waves just as a molecular lattice responds to light waves; the free electrons in the metal elements flow back and forth under the action of the alternating electric field, causing the elements to become oscillating dipoles similar to the oscillating molecular dipoles of the dielectric. In both cases, the relation between the effective dielectric constant ϵ of the medium, the density of the elements N (number per unit volume) and the dipole strength (polarizability α of each element) is approximately given by

$$\epsilon = \epsilon_0 + N\alpha \tag{2}$$

where ϵ_0 is the dielectric constant of free space.

There are two requirements which are imposed on the lattice structure. First, the spacing of the elements must be somewhat less than one wavelength of the shortest radio wave length to be transmitted, otherwise diffraction effects will occur as in ordinary dielectrics when the wavelength is shorter than the lattice spacing (X-ray diffraction by crystalline substances). Secondly, the size of the elements must be small relative to the minimum wavelength so that resonance effects are avoided. The first resonance occurs when the element size is approximately one half wavelength, and for frequencies in the vicinity of this resonance frequency the polarizability α of the element is not independent of frequency. If the element size is made equal to or less than one quarter wavelength at the smallest operating wavelength, it is found that α and hence ϵ in equation 2 is substantially constant for all longer wavelengths.

Since lenses of this type will effect an equal amount of wave delay at all wavelengths which are long compared to the size and spacing of the objects, they can be designed to operate over any desired wavelength band. For large operating bandwidths, the stepping process[2] is to be avoided, since the step design is correct only at one particular wavelength. Such unstepped lenses are thicker, but the diffraction at the steps is eliminated and a somewhat higher gain and superior pattern compared to a stepped lens is achieved. By tilting the lens a small amount, energy reflected from it is prevented from entering the feed line and a good impedance match between the antenna and the line can be maintained over a large band of frequencies.

Another way of looking at the wave delay produced by lattices of small conductors is to consider them as capacitative elements which "load" free

[2] The lens of Fig. 1 has 12 steps.

space, just as parallel capacitors on a transmission line act as loading elements to reduce the wave velocity. Consider a charged parallel plate air condenser with its electric lines of force perpendicular to the plates. Its capacity can be increased either by the insertion of dielectric material or by the insertion of insulated conducting objects between the plates if the objects have some length in the direction of the electric lines of force. This is because such objects will cause a rearrangement of the lines of force (with a consequent increase in their number) similar to that produced by the shift, due to an applied field, of the oppositely charged particles comprising the molecules of the dielectric material. The conducting elements in the lens may thus be considered either as portions of individual condensers, or as objects which, under the action of the applied field, act as dipoles and produce a dielectric polarization, similar to that formed by the rearrangement of the charged particles comprising a non-polar dielectric.[3] Either viewpoint leads to the delay mechanism observed in the focussing action of the artificial dielectric lenses to be described.

Experimental Models

We turn now to experimental exemplifications of lenses built in accordance with the principles outlined above.

(a) *Sphere Array*

One of the simpler shapes of conducting elements to be tried was the sphere. Figure 2 is a sketch of an array of conducting spheres arranged approximately in the shape of a convex lens. The spheres are mounted on insulated supporting rods; the microwave feed horn and receiver are shown at the right. The focal length is f, the radius of the lens "aperture" is y, the maximum thickness is x and not only the spacings s_1 and s_2 but also the size of the spheres are small compared to the wavelength. Rays A and B are of equal electrical length because ray A is slowed down or delayed in passing through the lens. Figure 3 is a photograph of the lens of Fig. 2; it also portrays a similar sphere array lens made of steel ball bearings supported by sheets of polystyrene foam[4] which have holes drilled in them to accept the spheres. In both cases the balls are arranged in a symmetrical lattice. It will be shown below that the polarizability α of a conducting

[3] Polar dielectrics have arrangements of charged particles which are electric dipoles even before an external electric field is applied; the field tends to align these and the amount of polarization (and hence the dielectric constant) that they exhibit depends upon temperature, since collisions tend to destroy the alignment. Non-polar (or hetero-polar) molecules have no dipole moment until an electric field is applied; the polarization of such materials (and of the artificial dielectrics we are discussing) is accordingly independent of temperature. See, for example, Debye, "Polar Molecules", Chap. III.

[4] Styrofoam (Dow). Because of its low density (1 to 2 pounds per cu. ft.), its contribution to the wave delay is negligible ($\epsilon_r = 1.02$).

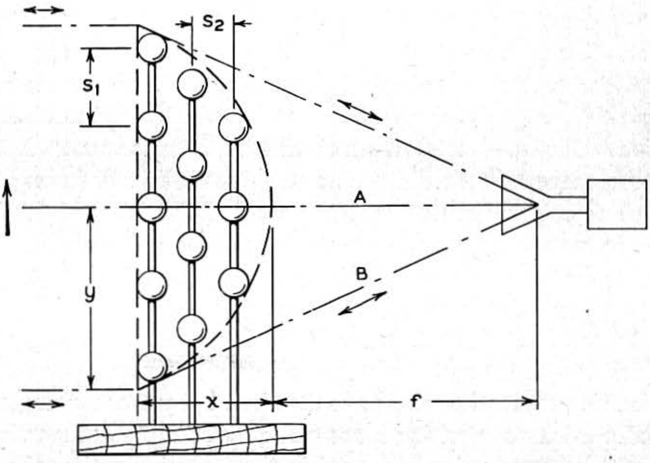

Fig. 2—Lattice of conducting spheres arranged to form a convex lens.

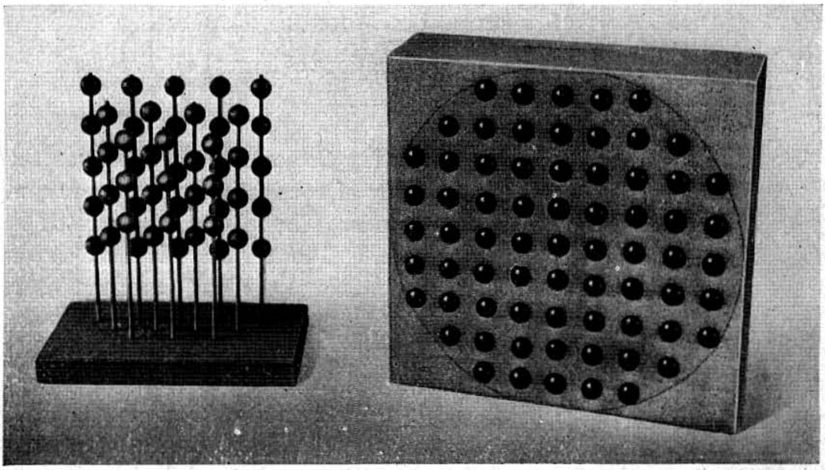

Fig. 3—Left: The sphere lens of Fig. 2. Right: Sphere lattice supported by foam sheets.

sphere of radius a is $4\pi\epsilon_0 a^3$, so that, for static fields, the relative dielectric constant is, from (2),

$$\epsilon_r = \epsilon/\epsilon_0 = 1 + 4\pi N a^3, \tag{3}$$

where N = number of spheres per unit volume.

For most dielectrics, the index of refraction is simply the square root of the relative dielectric constant. However, in the case of the sphere lens at microwaves eddy currents on the surface of the spheres prevent the magnetic lines of forces from penetrating them and it will be seen later that this effect causes the expected value of the index of refraction to be smaller than that determined by the usual equation

$$n^2 = \epsilon_r. \tag{4}$$

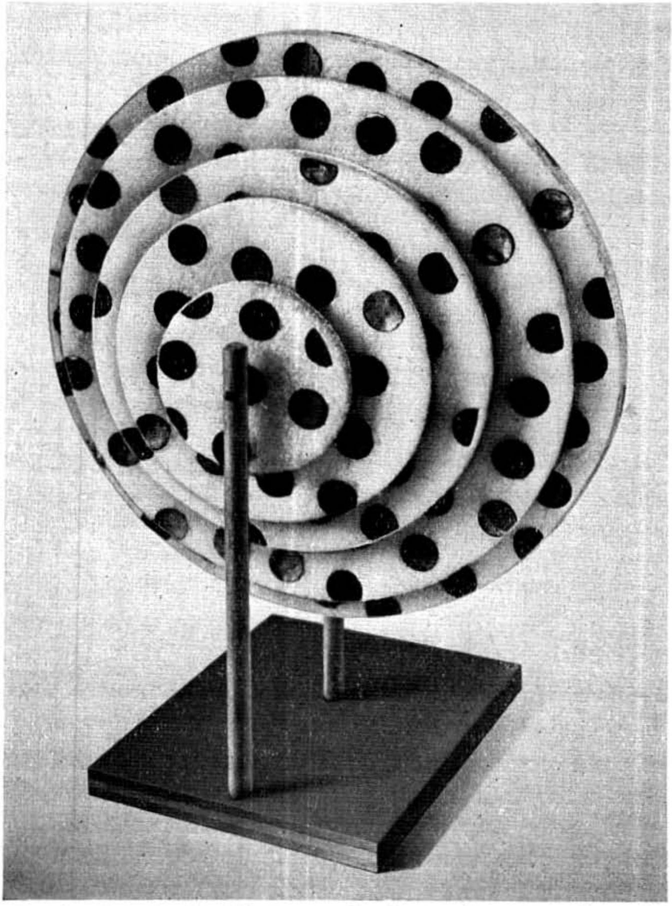

Fig. 4—Lattice of conducting disks arranged to form a plano-convex lens. Polystyrene foam sheets support the disks.

To avoid this effect the elements should be shaped so as not to alter the magnetic lines of force. This can be done by making their dimension in the direction of propagation of the impressed waves negligibly small.

(b) *Disk Array*

One accordingly arrives at the lens design shown in Fig. 4 in which the spheres are replaced by copper foil disks lying in planes parallel to the impressed E and H vectors. The supporting sheets are again polystyrene foam. The foil disks have negligible thickness so that the magnetic lines are unaffected as shown in Fig. 5. Equation (4) is therevore valid even at radio frequencies and the index of refraction of this material is obtainable from (4) and (2) with α equal to $\frac{16\,\epsilon_0\,a^3}{3}$, as shown in the last section.

(c) *Strip Array*

Both the sphere and disk type lenses have the advantage that they will perform equally well on horizontally or vertically polarized waves. If the lens is required to focus only one type of wave polarization the disks can be replaced by thin, flat, conducting strips extending in length in the direction

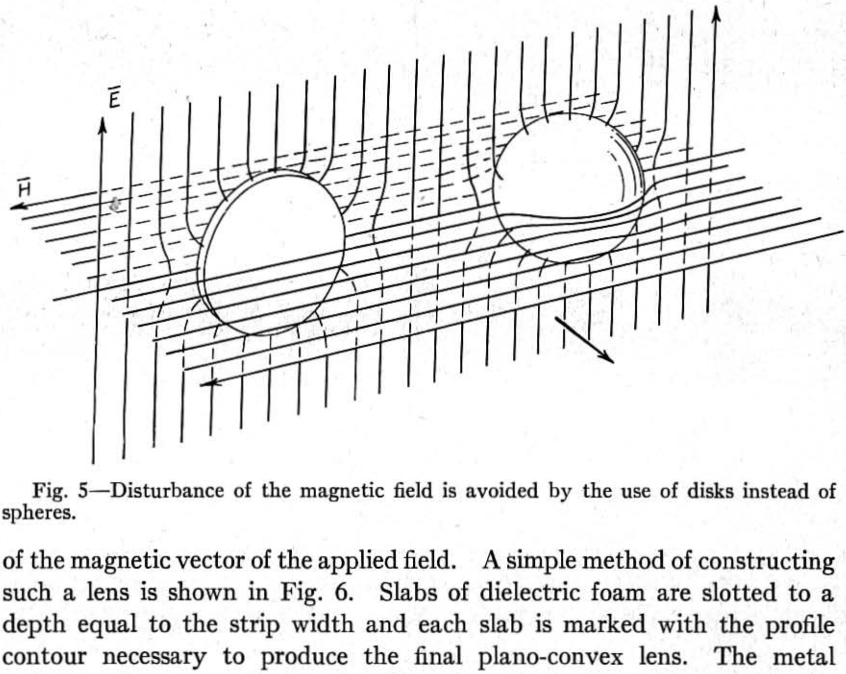

Fig. 5—Disturbance of the magnetic field is avoided by the use of disks instead of spheres.

of the magnetic vector of the applied field. A simple method of constructing such a lens is shown in Fig. 6. Slabs of dielectric foam are slotted to a depth equal to the strip width and each slab is marked with the profile contour necessary to produce the final plano-convex lens. The metal strips are then cut to the length indicated on the profile and inserted in the slots. With the strips in place the unit slabs are stacked on top of one another and held in a mounting frame to form the complete lens. Figure 7 shows one slab of a 10-foot strip type lens being assembled and Fig. 8 shows a six-foot square lens half assembled. Figure 9 shows directional patterns of a 3-foot diameter lens of this type made up of $\frac{3}{4}$ inch x .005 inch strips spaced $\frac{3}{8}''$ in the slabs and the slabs $1\frac{1}{2}''$ thick. The three patterns were taken over a 12% band of frequencies with the lens purposely illuminated by a low directivity feed at the focal point. This produced an almost uniform illumination across the aperture at all three wavelengths so that the side lobes were not too well suppressed. However, the deep minima in all three directional patterns indicate the relative absence of curvature of the emerging phase front; this signifies that the strip dielectric has a negligible variation of refractive index over the indicated wavelength band.

(d) *Sprayed Sheet Lenses*

The disk lens or strip lens can be constructed in the manner indicated in Fig. 10, which shows two lenses made by spraying conducting paint on thin dielectric sheets through masks. This results in round dot or square dot

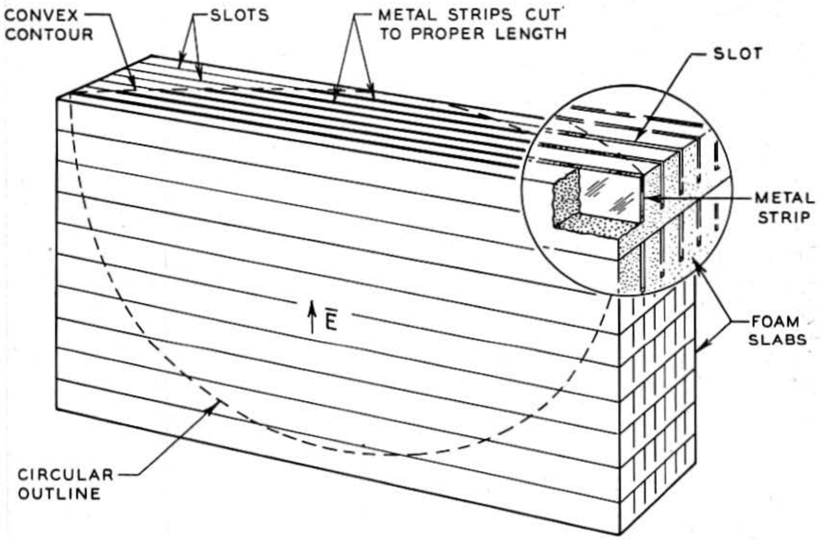

Fig. 6—Construction of a delay lens employing metallic strips as the delay elements.

patterns on the sheets and the size circle used on each sheet determines the three-dimensional contour when the sheets are stacked up to form the lens. Those in the photograph are spaced by wooden spacers as shown in the sketch of Fig. 11; for large lenses it would probably be preferable to cement the sheets to polystyrene foam spacers, thereby making a solid foam lens. Because of the small size of the elements, the lens on the left in Fig. 10, was effective at wavelengths as short as 1.25 cm, in addition to longer wavelengths (3, 7 and 10 cm). The lens of Fig. 12 was made by metal spraying metallic tin directly on circular foam slabs through masks, and the foam disks then cemented together.

Strip lenses can also be constructed in this way. The lens in Fig. 13,

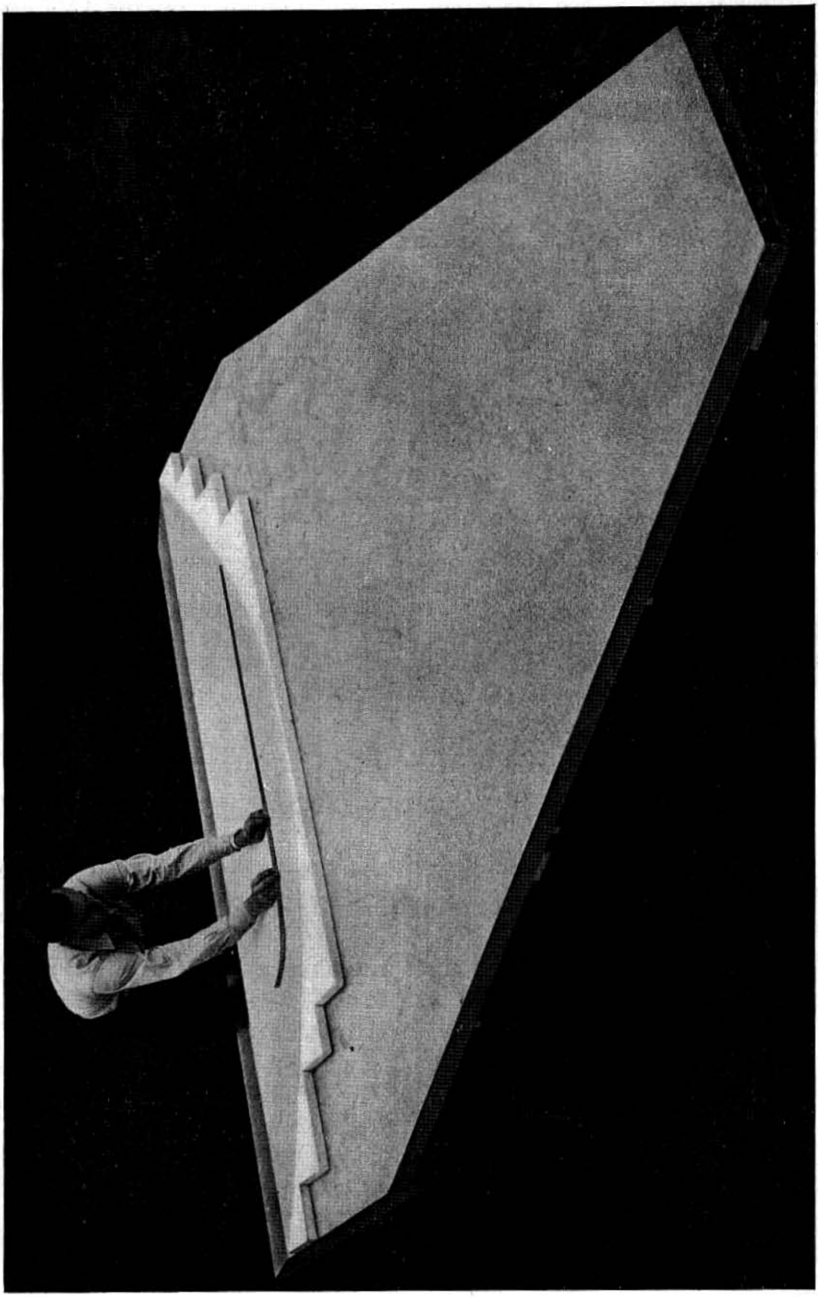

Fig. 7—Inserting metal strips into a profile plate of a 10-foot strip lens.

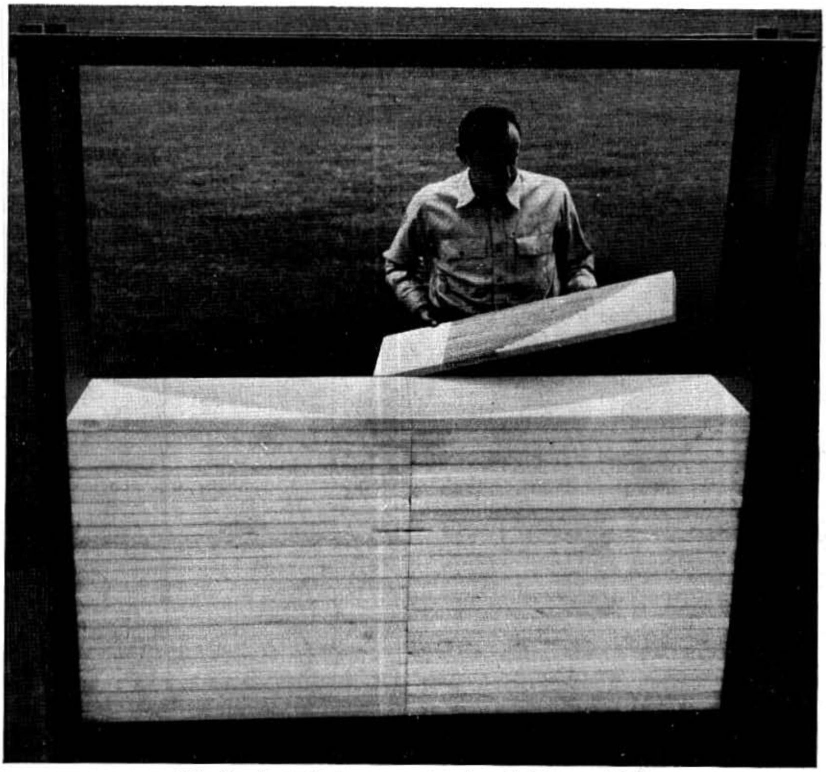

Fig. 8—A six-foot square strip lens half assembled.

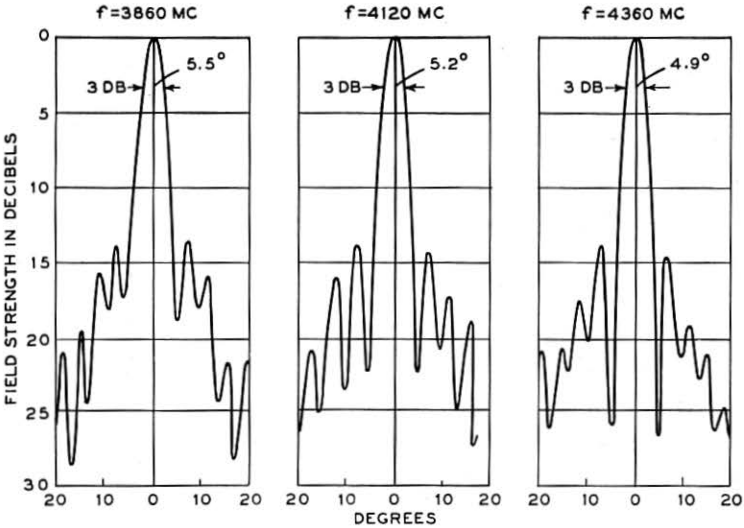

Fig. 9—Directional patterns of a 3-foot diameter lens at several frequencies.

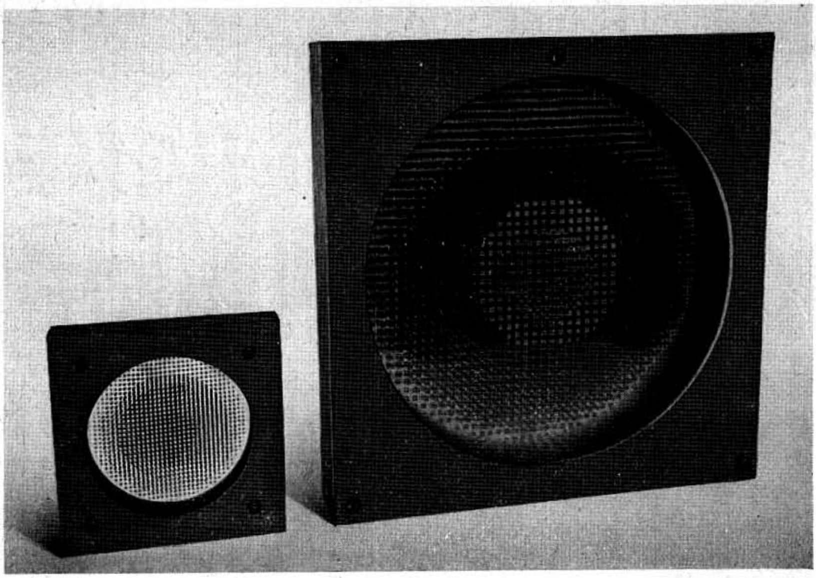

Fig. 10—Lenses constructed by spraying conducting paint on cellophane sheets.

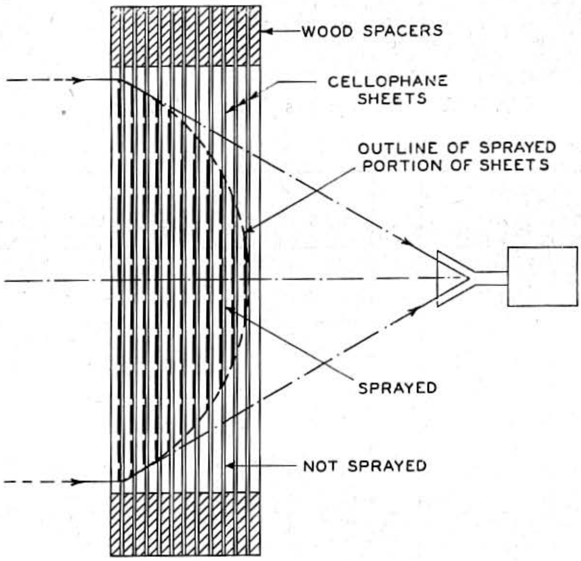

Fig. 11—Construction details of the lenses of Fig. 10.

was made by affixing copper foil strips to cellophane sheets and stacking the sheets with no spacing other than the sheets themselves between them.

This extremely close spacing and the staggered arrangement of strips correspond to a heavy capacitative loading and introduced so much delay per unit length that the measured effective dielectric constant of this lens proved to be 225 (i.e. the index of refraction was 15). Because of such a high dielectric constant the reflection losses at the surface of this lens are high,

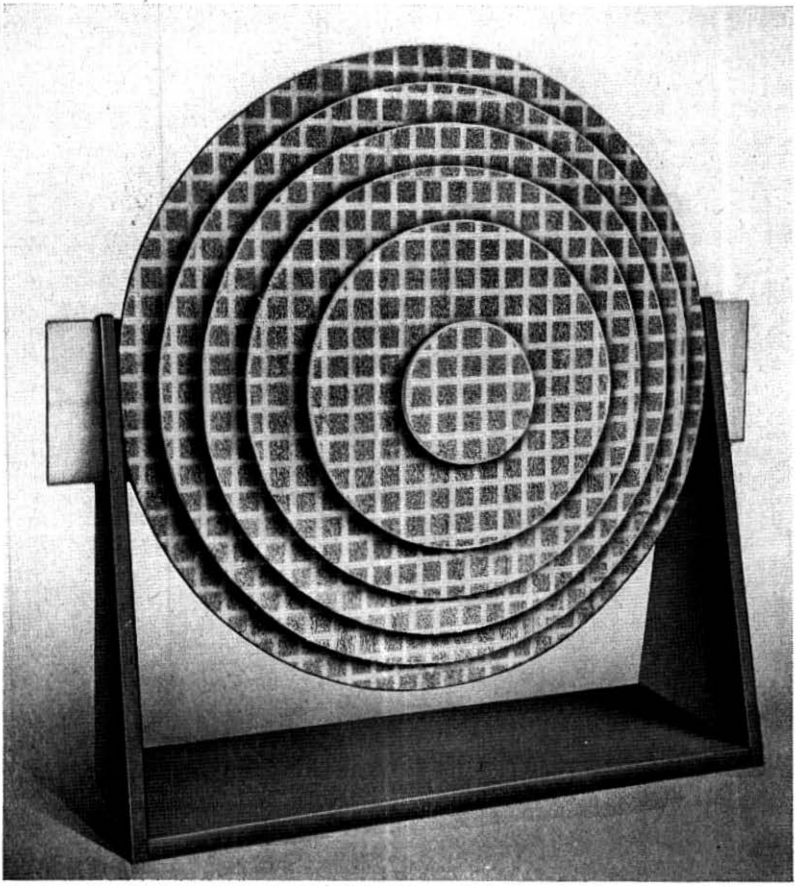

Fig. 12—Lens formed by metal-spraying tin directly onto polystyrene foam sheets.

so that, for efficient operation, tapered or quarter wave matching sections on each surface would be necessary.

(e) *Frequency Sensitive Delay Lenses*

When the conducting elements approach a half wavelength in their length parallel to the electric vector, resonance effects occur and the artificial dielectric behaves like an ordinary dielectric near its region of anomalous dispersion.[5] The index of refraction of an artificial dielectric using $\frac{3}{4}''$ metallic elements would increase rapidly as the wavelength approached $1\frac{1}{2}''$ until, at $\lambda = 1\frac{1}{2}''$ the dielectric would be opaque. At still lower wavelengths, the material would appear to have an index of refraction less than one.

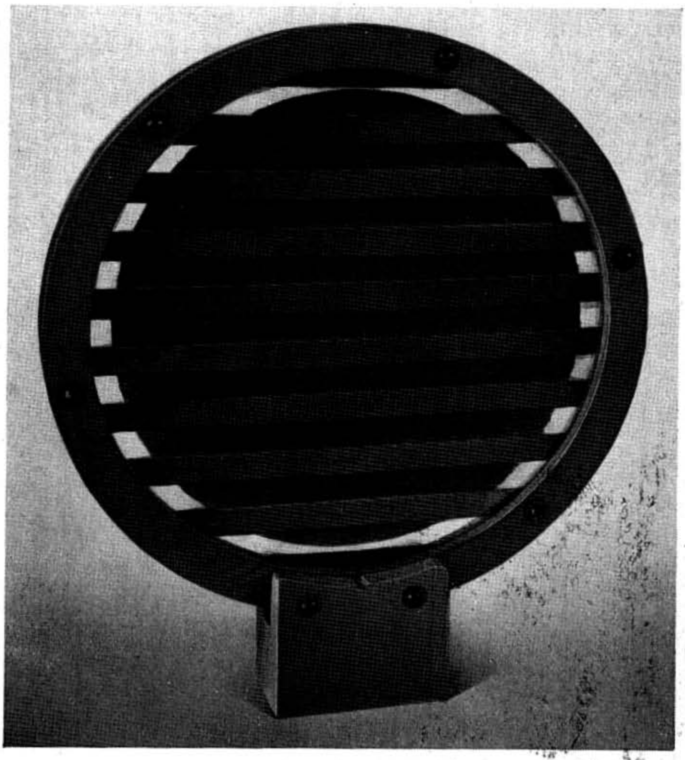

Fig. 13—Closely spaced strip construction comprising copper foil strips affixed to cellophane sheets. Juxtaposition of the sheets yielded an effective index of refraction of 15.

Elements, such as rods, which are $\lambda/2$ long, have a very broad resonance band, and the region of anomalous dispersion in a dielectric utilizing such

[5] Anomalous dispersion occurs in optical substances when the frequency of the incident light wave approximates one of the vibrational resonance frequencies of the molecule. On the long wavelength side of this resonance region the index of refraction is greater than one and increases rapidly as the resonance wavelength is approached. Dispersion, which is the change of index of refraction with frequency, is therefore very high, but it is the "normal" type of dispersion. At resonance, the absorption of the wave is high and the substance becomes opaque. At still shorter wavelengths, the resonance phenomenon acts to make the index of refraction less than its long wavelength value, often less than one, and the index again varies rapidly with wavelength, but because this is an "abnormal" type of dispersion, it is called the region of anomalous dispersion.

rods is very large, i.e., the dispersion is not very great. If it is desired to have a highly dispersive material, this band can be considerably reduced by the process of tilting the rods so that they are more nearly perpendicular to the electric vector. They thereby become "loosely coupled" to the incident wave and acquire a higher Q. Some unsymmetrical arrangement such as that shown in Fig. 14 is necessary to insure that the radiation damping of the elements is actually reduced, since a uniform tilt of all the elements would allow the array to radiate, unhindered, a wave polarized parallel to the elements. Measurements of the index of refraction of a dielectric made up of successive arrays of rods arranged as in Fig. 14 are

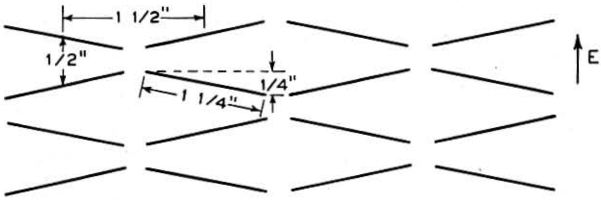

Fig. 14—Array of metal rods to produce a narrow dispersion band.

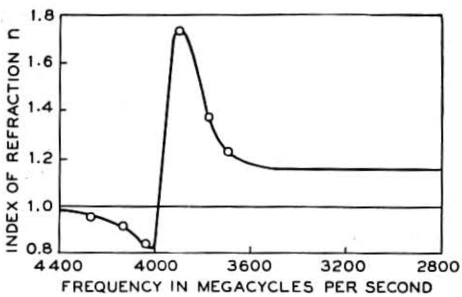

Fig. 15—Measured index of refraction of a metallic dielectric comprising the tilted rods of Fig. 14.

given in Fig. 15, and one observes a marked similarity to the behavior of the index of refraction of dielectrics in the region of their anomalous dispersion. Because of the rapid change of n with wavelength, such material may be useful as a means of separating narrow band radio channels by the use of prisms or by lenses having several feed horns located at the optimum focal points for the various frequency bands involved.

Microwave Repeater Antenna

For radio relay applications, there are three electrical characteristics which antennas should possess. The first is high gain (effective area), as this will reduce the path loss and accordingly the requirements on transmitter power. The second is good directional qualities so as to minimize interference from outside sources and also interferences between adjacent antennas. The third is a good impedance match so that reflections between the antenna and the repeater equipment will not distort the transmitted signal. These characteristics should preferably be attainable without the imposition of severe mechanical or constructional requirements.

The shielded lens type of antenna (lens in the aperture of a horn) lends itself well to repeater work because of its moderate tolerance requirements, its good directional properties associated with the shielding, and the desirable impedance characteristic obtainable by tilting the lens. The delay lens offers the additional advantage of broad band performance with the consequent possibility of operating on several bands widely separated in wavelength. In this section, construction details and performance of a 6-foot square aperture strip type delay lens will be discussed.

(a) *Design of the Artificial Dielectric*

The operating frequency band envisioned for this antenna was 3700 to 4200 megacycles ($\lambda = 7.15$ to 8.1 cm), and to keep the element length sufficiently well removed from the half-wave resonant length a value of $\frac{3}{4}''$ for the strip width was chosen. The index of refraction of solid polystyrene is approximately 1.61 and this introduces a reflection loss (mismatch loss) at each surface of 0.225 decibels. To reduce this loss to 0.18 db. the artificial dielectric was designed to have an n of 1.5 as this still did not impart too great a thickness to the lens. A combination of strip spacings which yields an n of 1.5 was found to be $\frac{3}{8}''$ in the horizontal direction and $1\frac{5}{16}''$ center to center spacing in the vertical direction as shown in Fig. 16. The construction method of Fig. 6 was used which involved inserting .002″ copper strip into slots cut in foam sheets.

(b) *Lens Design*

Several lens shapes were possible: (1) bi-convex, (2) plano-convex with the flat side toward the feed, and (3) plano-convex with the curved side toward the feed. For a given thickness and therefore weight of lens, the third possibility produces the shortest over-all structure of lens plus horn feed, and it was accordingly selected. The curved side is then a hyperboloid of revolution as shown in Fig. 17 and for the chosen focal length of 60″, the profile, as calculated from the equation shown for $n = 1.5$, reaches a maximum thickness of 16″. To eliminate the reflection from the lens into the feed, a lens tilt could have been employed. It was found that a quarter wave offset of one half of the lens relative to the other half could accomplish this same purpose, because reflected rays from one half of the lens then undergo a one half wavelength longer path in returning to the feed and the reflections from the two halves cancel. As this process, however, is completely effective only at one frequency, the final lens design employed both a

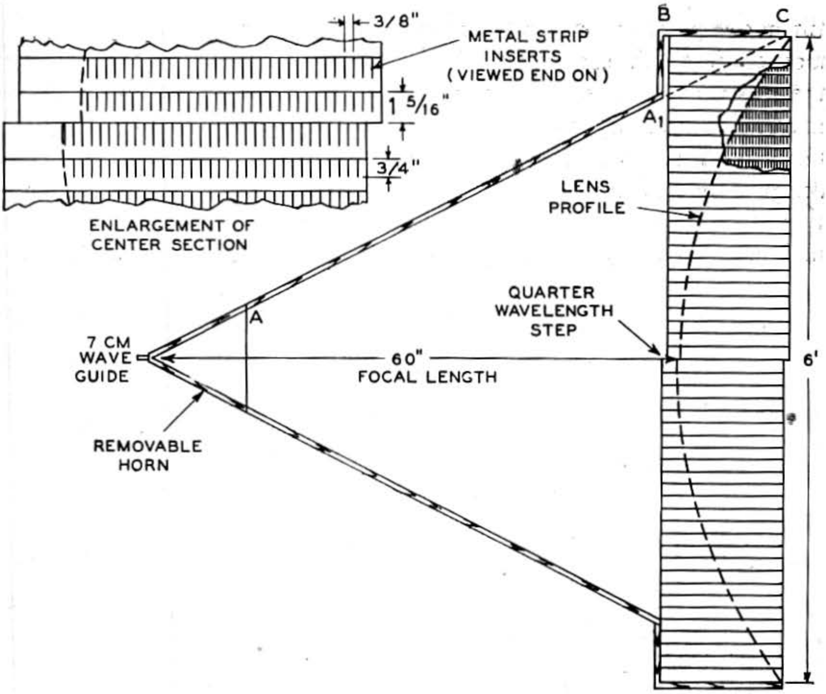

Fig. 16—Construction of a six-foot square shielded delay lens for repeater application. The wooden horn has a metal interior surface.

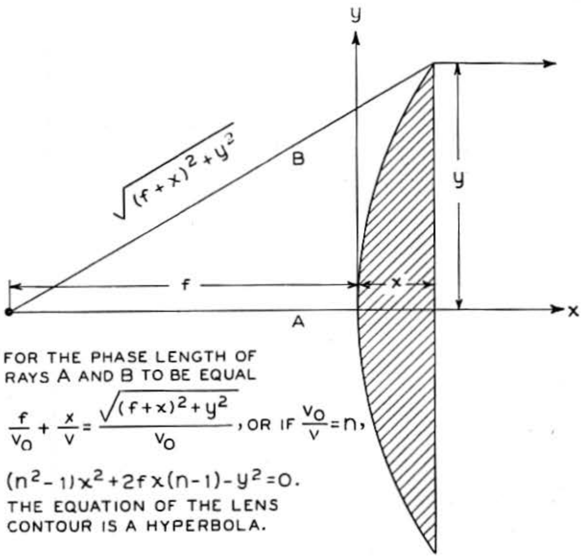

Fig. 17—Profile equation for a delay lens.

tilt in one plane and a quarter wave offset in the other plane. To utilize most efficiently the space afforded the antenna on the top of the relay tower, the lens aperture was made square. Since an unstepped lens has, by its nature, a circular aperture, the four corners of the horn aperture must be filled in with lens material as sketched in Fig. 18. The step height is designed for midband wavelength and in the present case follows the equation of Fig. 17 with the focal length reduced from the value used for the main lens section by $K\lambda/(n - 1)$ where K is an integer. That integer K is

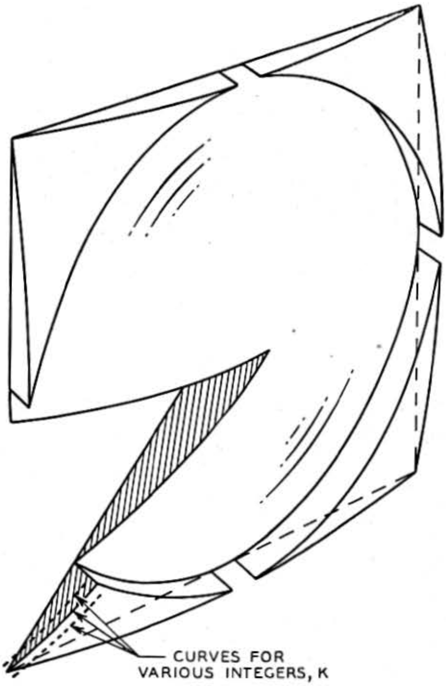

Fig. 18—Filling in the four corners of a square horn aperture with lens material.

selected which brings the step profile nearest the horn corners as indicated in Fig. 18.

(c) *Horn Shield*

Lenses can be energized by means of a small feed horn placed at the focal point, but to obtain the best directional properties a full metallic shield extending from the wave guide feed up to the sides of the lens should be employed. The use of foam slabs in the construction of the delay lens prevents this shield from extending completely up to the lens as indicated by the extension of A-A_1 (shown dotted) in Fig. 16. Optical formulas indicate, however, that for such large apertures, the amount of diffraction (spreading of the waves outside of the pyramid formed by the dotted lines) will be small, and it is only necessary that the sides AA_1, A_1B and BC all be conducting to insure good back lobe suppression and other desired properties associated with a horn shield.

(d) *Performance*

The gain of this antenna over an isotropic radiator is plotted in Fig. 19. The top curve is the theoretical gain of a uniform current sheet of the same area (6′ x 6′), the lower curve the gain of a 6′ x 6′ area having 60% effective area. The points, which fall approximately on the lower curve, are the experimentally measured gain values of this antenna at the frequencies

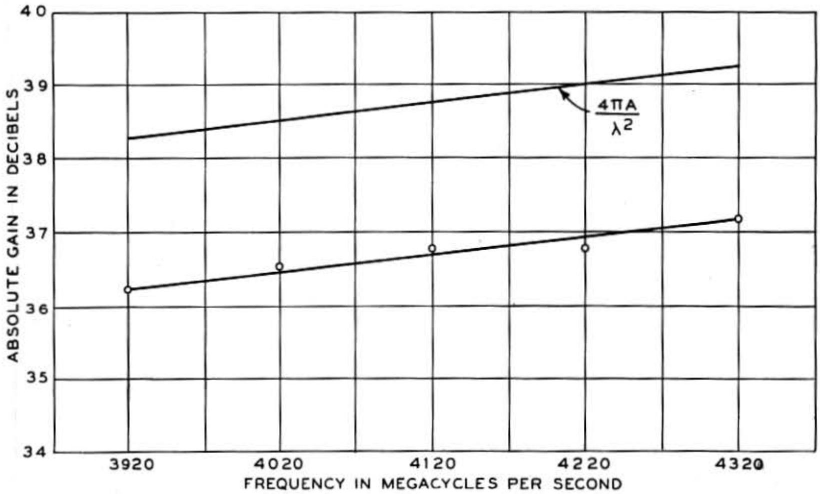

Fig. 19—Measured gain characteristics of the six-foot square shielded lens of Fig. 16. The lower line indicates 60% effective area and the circles are experimental points.

indicated. The constant percentage effective area indicates that the index of refraction of the lens material remains quite constant over the indicated frequency band. In contrast to this, the 10′ x 10′ metal plate lenses[1] ($n < 1$) exhibit, at the band edges, a falling off of $1\frac{1}{2}$ decibels from midband gain for a 10% wavelength band.

The magnetic plane pattern of the lens when fed by a 6-inch square feed horn is shown in Fig. 20. The remarkable symmetry of the minor lobes in Fig. 20 and also in Fig. 9 shows that the phase fronts of the waves radiated by these antennas are very accurately flat. This result emphasizes the tolerance advantages of the lens over the reflector. It is believed that the

[1] Loc. cit.

further improvement in pattern of this lens over previous wave guide type metal lenses is attributable to the absence of steps, as they tend to introduce diffraction effects. The symmetry of the pattern also indicates a high degree

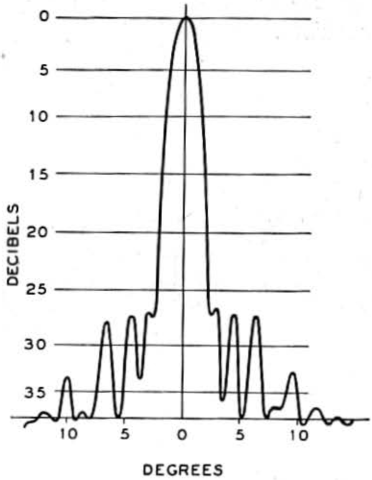

Fig. 20—Directional pattern of the lens of Figs. 8 and 16 when fed with a six-inch square electromagnetic horn.

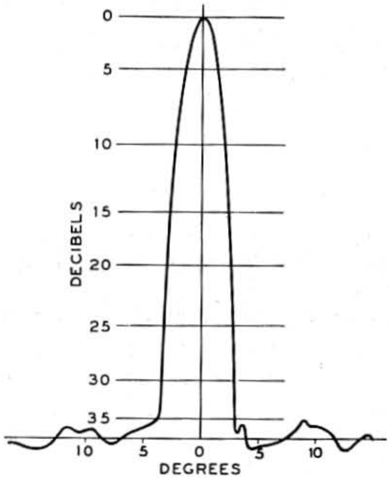

Fig. 21—Directional pattern of the same lens when enclosed with a full horn shield.

of homogeneity of the dielectric (the lens is 16 inches thick); this is a property not always shared by ordinary dielectrics such as polystyrene.

When a full horn shield is used, the illumination across the aperture has a very strong taper (somewhat stronger than a cosine taper because of the wide flare angle of the horn). This results in a very effective suppression of the close-in side lobes as shown in Fig. 21. In the vertical plane, the side lobes are not very well suppressed because the illumination is only slightly tapered. However, for repeater work, lobes in the vertical plane are not as objectionable as lobes in the horizontal plane because interfering signals generally originate from other stations lying in the horizon plane.

The impedance match of the shielded lens antenna is affected by the discontinuity of the wave guide at the expanding horn throat and by whatever energy is reflected back into the feed line from the lens. By tuning means, the throat mismatch can be held to less than 0.2 db SWR (1.02 V.S.W.R.) over the 500 mc band and by a combination of lens tilt and quarter-wave step in the lens the SWR due to the energy reflected from the lens was also held to less than 0.2 db over the band.

PART II—THEORETICAL CONSIDERATIONS

We turn now to a consideration of the electromagnetic theory underlying the operation of artificial dielectrics. If the polarizability of the individual conducting element employed is known, equation (2) will permit a calculation of the effective dielectric constant. Before proceeding to this, however, a brief review of dipole moment, dipole potential, and dielectric polarization will be given (MKS units).

Dipole Moment, Potential and Polarization

Two charges, $+q$ and $-q$, displaced a small distance from one another, constitute an electric dipole. If the vector joining them is called $\boldsymbol{ds}$, the dipole moment is defined as

$$\boldsymbol{m} = q\, \boldsymbol{ds}. \tag{5}$$

The potential V, at any point, due to a point-charge q is defined as

$$V = q/4\pi\epsilon r, \tag{6}$$

where r is the distance from the point in question to the charge q. The potential V due to a dipole of moment $\boldsymbol{m}$ is

$$V = \frac{|m|}{4\pi\epsilon r^2} \cos\theta, \tag{7}$$

where θ is the angle between $\boldsymbol{r}$ and $\boldsymbol{m}$.

A conducting object becomes polarized when placed in an electric field. Its dipole moment $\boldsymbol{m}$ depends upon the field strength $\boldsymbol{E}$ and upon its own polarizability α:

$$\boldsymbol{m} = \alpha \boldsymbol{E}. \tag{8}$$

If there are N elements per unit volume, the polarization $\boldsymbol{P}$ of the artificial dielectric is

$$\boldsymbol{P} = N\alpha \boldsymbol{E}. \tag{9}$$

But $\boldsymbol{P}$ is related to the displacement vector $\boldsymbol{D}$ and the dielectric constant as follows:

$$\boldsymbol{D} = \epsilon \boldsymbol{E} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}, \tag{10}$$

so that $\epsilon = \epsilon_0 + N\alpha$ which is equation (1). A knowledge of α thus permits a determination of ϵ. We obtain α from (8) by first finding the dipole moment $\boldsymbol{m}$ of the particular shape of element when immersed in a uniform field $\boldsymbol{E}$.

Calculation of Dielectric Constants of Artificial Dielectrics[6]

(1) *Conducting Sphere*

Consider a perfectly conducting sphere immersed in an originally uniform field of potent'al

$$V = -Ey = -Er \cos \theta. \tag{11}$$

The free charges on the sphere are displaced by the applied field and it thereby becomes a dipole whose moment $\boldsymbol{m}$ we wish to determine. The external potential field is the sum of the applied potential and the dipole potential, and from (7) and (11) we have

$$V_{out} = -Er \cos \theta + \frac{m \cos \theta}{4\pi\epsilon_0 r^2}. \tag{12}$$

The internal field is zero because the sphere is conducting. At a boundary between two dielectrics, there is the requirement[7]

$$V_{\text{outside}} = V_{\text{inside}}. \tag{13}$$

Equation (13) gives, at $r = a$ (the radius of the sphere),

$$-Ea \cos \theta + \frac{m \cos \theta}{4\pi\theta_0 a^2} = 0, \tag{14}$$

or

$$m = 4\pi\epsilon_0 Ea^3, \tag{15}$$

the dipole moment of the sphere. From (8) we see that the polarizability of the conducting sphere is accordingly $4\pi\epsilon_0 a^3$, from which equation (3) follows.

(2) *Magnetic Effects of a conducting sphere array*

The above calculations on a conducting sphere assume an electrostatic field. At microwaves, the rapidly varying fields induce eddy currents on the surface of the sphere which prevent the magnetic lines of force from penetrating the sphere. The magnetic lines are perturbed as shown in

[6] The author is indebted to Dr. S. A. Schelkunoff for the polarizability formulas given in this memorandum.

[7] Smythe, "*Static and Dynamic Electricity*", McGraw-Hill, 1939, p. 19.

Fig. 5. Now a conducting sphere in an alternating magnetic field is equivalent to an oscillating magnetic dipole,[8] and we should observe an effective permeability for a sphere array.[9] The magnetic dipole field is, however, opposed to the inducing field, in other words, the dipole moment is negative. Smythe[7] shows that the magnetic polarizability of a conducting sphere of radius a in a high frequency field is

$$\alpha_m = -2\pi\mu_0 a^3. \tag{16}$$

The effective relative permeability of an array of N spheres per unit volume is therefore

$$\mu_r = 1 - 2\pi a^3 N. \tag{17}$$

The index of refraction was given in equation (4) as the square root of the dielectric constant. This is strictly true only when the permeability of the dielectric is unity. We have seen above, however, that the sphere array at microwaves possesses an effective permeability given by (17), and therefore (4) is not valid. The correct expression for n is

$$n = \sqrt{\mu_r \epsilon_r}, \tag{18}$$

and the effective refractive index of a sphere array is accordingly

$$n = \sqrt{(1 - 2\pi N a^3)(1 + 4\pi N a^3)}, \tag{19}$$

which is smaller than that given by (3) and (4). The disk and strip arrays, besides being lighter, avoid the diminishing effect on the refractive index caused by the perturbing of the magnetic lines.

(3) *Conducting Circular disk*

The determination of the dipole moment of a disk involves the use of ellipsoidal coordinates and will not be carried through here. Smythe[7] gives an expression for the torque on a flat disk of radius a in a uniform field in Gaussian units. In M.K.S. units, his formula becomes

$$T = 4\pi\epsilon_0 \frac{2a^3 E^2 \sin 2\theta}{3\pi} = 4\pi\epsilon_0 \frac{2a^3 E^2 \, 2 \sin\theta \cos\theta}{3\pi}$$

$$= \left[\frac{16a^3 \epsilon_0}{3} (E \sin\theta)\right][E \cos\theta]. \tag{20}$$

The first bracket represents the dipole moment, the second the field, and the product gives the torque. When the plane of the disks is parallel to E $\sin\theta$ is one, and from $\boldsymbol{m} = \alpha \boldsymbol{E}$, we have

$$\alpha = \frac{16\epsilon_0 a^3}{3}. \tag{21}$$

[8] T. S. E. Thomas, *Wireless World*, Dec. 1946, p. 322.
[9] L. Lewin, *Jour. I. E. E.*, Part III, Jan. 1947, p. 65.
[7] Loc. Cit. Eq. 14, p. 397.
[7] Ibid, eq. 7, p. 163.

so that, from (2),

$$\epsilon_r = 1 + \tfrac{16}{3} N a^3 \tag{22}$$

(4) *Strips*

The calculation of the dipole moment of a thin conducting strip as used in the strip lens of Fig. 15 involves two dimensional elliptic coordinates and will also be omitted here. Again, the torque is given in Smythe[7] for an elliptic dielectric cylinder, from which we obtain,

$$\alpha = \frac{\pi \epsilon_0 s^2}{4}, \tag{23}$$

where s is the strip width, so that

$$\epsilon_r = 1 + \frac{\pi}{4} s^2 n, \tag{24}$$

where n is the number of strips per sq. unit area looking end on at the strips.

(5) *Validity of the polarizability equations*

Equation (2), which expresses the dielectric constant to be expected from an array of N elements each having a polarizability of α, was derived (by (8), (9) and (10)) by assuming that the field acting on an element, and tending to polarize it, was the impressed field $\boldsymbol{E}$ alone. This is a satisfactory assumption when the separation between the objects is so large that the elements themselves do not distort the field acting on the neighboring elements. Such is not the case when the value of ϵ_r exceeds 1.5 or thereabouts. For the usually desired values of ϵ_r of 2 or 3 it is thus seen that the above formulas such as equations (22) and (24) will yield only qualitative results and that the exact spacings of the elements to produce a desired refractive index will have to be determined by experimental methods.

For lattices having 3-dimensional symmetry, an improvement over equation (2), which takes into account not only the impressed field $\boldsymbol{E}$ but also the field due to the surrounding elements, is the so-called Clausius-Mosotti equation:

$$\frac{\epsilon - \epsilon_0}{\epsilon + 2\epsilon_0} = \frac{N\alpha}{3\epsilon_0} \tag{25}$$

This, along with a similar expression for the permeability [replacing (17)], would permit a fairly accurate determination from (18) of n for the conducting sphere array.

[7] Ibid, eq. 6, p. 97.

Resonance Effects

It was stated earlier that the size of the elements should be small relative to a half wavelength in order for the refractive index to be independent of frequency. A qualitative idea of this criterion can be obtained by an elementary analysis of forced oscillations of dipoles. It is known that a dielectric medium which is composed of elements that resonate under the action of an alternating electric field, such as atoms having bound electrons, will exhibit a dielectric constant which varies with frequency:[10]

$$\epsilon_r = 1 + \frac{k}{f_0^2 - f^2}, \tag{26}$$

where f_0 is the frequency of resonance of the element, f the frequency of the incident radiation and k a proportionality constant. Thus when f is small relative to f_0, ϵ_r is practically independent of f.

As a means of estimating the change in refractive index with frequency of metal delay lenses, we consider a specific example: Let $n = 1.50$ when the elements are $\lambda/4$ in length, i.e. when $f^2 = \frac{1}{4}f_0^2$, then the last term of (26) equals 1.25. Decreasing f by 20% reduces n from 1.50 to 1.46. Thus the change in n from midband to the edges of a $\pm$ 10% band is about .02. From this, at 7 cm, the phase front, even for a lens 30″ thick, should remain plane to within $\pm\frac{1}{10}$ wavelength over this 20% band of wavelengths. If the members had been made $\frac{1}{8}$ wavelength long, the variation in n from the design frequency all the way down to D.C. would have amounted to only 1.2%.

Summary

A *metallic dielectric* is constructed by arraying conducting elements in a three-dimensional lattice structure. For electromagnetic waves whose wavelength is long compared to the size and spacing of the elements, this structure displays an effective dielectric constant and index of refraction which is sensibly constant over wide frequency bands. Lenses can be designed according to these principles which will focus microwaves and longer radio waves as a glass lens focusses light waves. Such lenses have the advantage of broad-band performance over the earlier waveguide type metal lenses and they retain the advantages of light weight over dielectric lenses. As microwave antennas, they are superior to parabolic dish reflectors from the standpoint of warping and twisting tolerance, profile tolerance, directional properties and impedance match. By eliminating the steps in the lens, the directive patterns are made cleaner and an increase in absolute

[10] See for example, Joos, *Theoretical Physics*, Blackie & Son, Book 4, Chapter 4.

gain results. Because of the broad-band properties of the artificial dielectric, this improved gain can be maintained over a very wide band of frequencies. The lenses can be built to focus waves of any polarization and, if desired, the dielectric can be designed to exhibit strong dispersion. Theoretical calculations of the expected dielectric constant are in fairly good agreement with experiment for values less than 1.5; for higher values an accurate determination of the true value must be obtained experimentally.

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