Experiment 8 Interferometers

> The true worth of an experimenter consists in his pursuing not only what he seeks in his experiment, but also what he did not seek.
>
> Claude Bernard (1813-1878)

OBJECTIVES

To examine the operation of several kinds of interferometers.

THEORY

Generically, an interferometer is a device for producing interference between two or more waves. There are numerous types with various features, but only two distinctly different strategies. The first deliberate interferometer, Young's two-slit experiment, divided a wavefront spatially and then recombined the parts. The other approach is to use some sort of partial reflector to divide the amplitude of the incident wave into separate beams which are eventually rejoined. Either method can make use of multiple beams. We will study some examples of each class.

As already mentioned, Young's two-slit interference experiment is the archetype of wavefront division. The geometry is shown in Fig. 8-1. A plane wave hits two slits, where it is divided into two diverging wavefronts by diffraction. The wavefronts emerging from the slits overlap and interfere in the region beyond the slits. The phase difference between the waves arriving at a given point is determined by the path difference between the points, leading to maxima at angles θ given by

$$d \sin\theta = n\lambda \qquad (8\text{-}1)$$

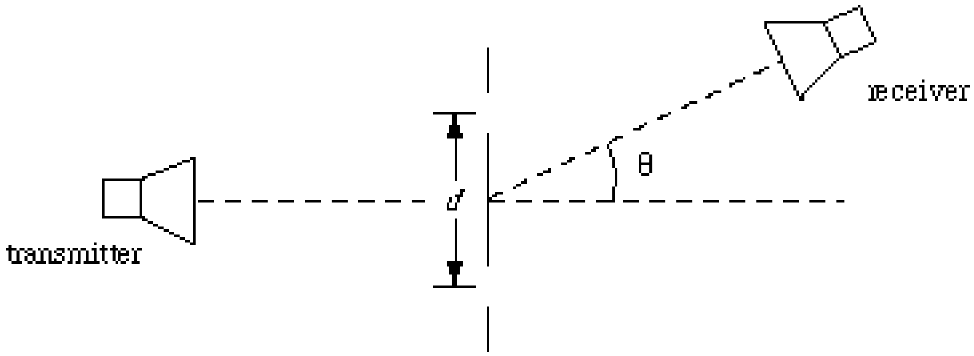

Fig. 8-1 Diagram of two-slit interference experiment.

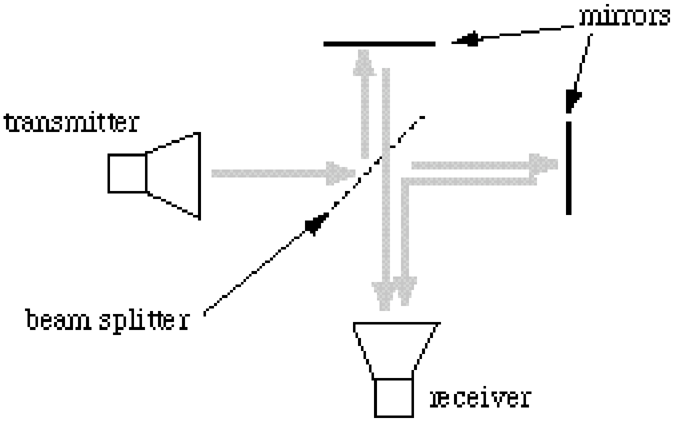

Fig. 8-2. Schematic diagram of a Michelson interferometer.

for wavelength λ and slit separation d. In a later experiment we will examine a generalization of this relation to a multi-beam interferometer (many slits), but for now this is the only result we need.

The Michelson interferometer, shown in Fig. 8-2, is a simple example of a two-beam amplitude-splitting instrument. The incoming wave is partially reflected and partially transmitted by a beam splitter. The reflected wave goes to one mirror and the transmitted wave goes to the other. After reflection from the mirrors, the beams are combined again at the beam splitter and proceed to the detector. If the two path lengths are not equal the returning waves may combine either in phase or out of phase, resulting in an increase or decrease of the received amplitude. It is easy to see that moving either mirror by a distance x changes the path length by $2x$. Accordingly, we should expect to see successive minima (or maxima) at the receiver when we move either mirror by half a wavelength.

If we arrange two partially transmitting plates as shown in Fig. 8-3, we can cause the beam to bounce back and forth many times, creating a multi-beam amplitude-division instrument called a Fabry-Perot interferometer. The analysis is simplified by noting that the net effect of the multiple reflections is to create a standing wave by superposing two counter-propagating waves. We feed energy in through one plate, and detect the energy that comes out through the other plate. For a quantitative analysis, we need to derive an expression for the amplitude of the electromagnetic wave emerging from the second plate, in terms of the amplitude of the input wave.

For a single plate, the incident, reflected and transmitted electric field amplitudes are related by reflection and transmission coefficients r and t defined as follows:

$$E_{ref} = rE_{inc} \qquad E_{trans} = tE_{inc} \qquad (8\text{-}2)$$

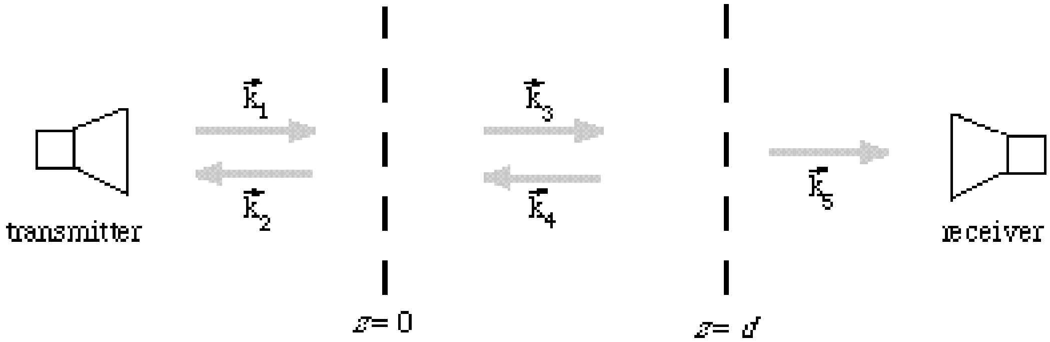

Fig. 8-3 Geometry of a Fabry-Perot interferometer

(We have assumed normal incidence, and deal only with the vector magnitudes so that r and t are scalars. We also neglect the phase shift due to the plate.) If, in addition, there is no conversion of electromagnetic energy to heat, conservation of energy requires that

$$r^2 + t^2 = 1 \tag{8-3}$$

At any point in space, we can write the scalar fields in the complex representation as

$$E_j = E_{j0} e^{i(\pm kz + \omega t)} \tag{8-4}$$

with

$$k = \frac{2\pi}{\lambda} \tag{8-5}$$

The plus sign indicates travel to the left in Fig. 8-3. Referring again to Fig. 8-3, we can relate the reflected and transmitted waves at $z = 0$ by

$$E_{20} = tE_{40} + rE_{10} \tag{8-6}$$

$$E_{30} = tE_{10} + rE_{40} \tag{8-7}$$

and similarly at $z = d$

$$E_{40} e^{ikd} = rE_{30} e^{-ikd} \tag{8-8}$$

$$E_{50} e^{-ikd} = tE_{30} e^{-ikd} \tag{8-9}$$

Solving for E_{50} in terms of E_{10} we obtain, after some fuss,

$$E_{50} = \frac{t^2}{1 - r^2 e^{-2ikd}} E_{10} \tag{8-10}$$

The quantity we actually measure is the transmitted intensity, which is proportional to $|E_{50}|^2$. Doing the algebra, we arrive at the ratio of transmitted to incident intensities

$$\frac{|E_{50}|^2}{|E_{10}|^2} = \frac{t^4}{1 + r^4 - 2r^2 \cos \frac{4\pi d}{\lambda}} \tag{8-11}$$

where we have used the definition (8-5) of k in terms of wavelength λ. This is the result we need to interpret the experiment.

The right hand side of Eq. 8-11 is plotted in Fig. 8-4 for several values of the reflection coefficient r of the plates. Note that the maxima and minima of transmission occur at half-wavelength intervals, as one could also see directly from the periodicity of $\cos(4\pi d/\lambda)$. As r increases, the amount of transmission between maxima decreases sharply, and the peaks become narrower. At optical frequencies r can be made very nearly one, resulting in very sharp peaks indeed. If several wavelengths are present in the incident illumination, each one will produce a peak at a particular set of separations and the device is useful as a high resolution spectrometer.

EXPERIMENTAL PROCEDURE

Our electromagnetic wave generator is a microwave klystron, connected to a horn antenna to produce approximately plane waves with wavelength about 3cm. The electric field of

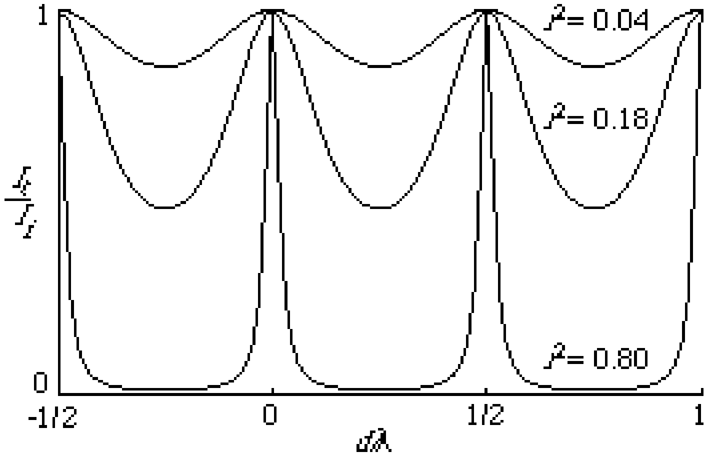

Fig. 8-4 Ratio of transmitted to incident intensity as a function of d for the Fabry-Perot.

the waves is parallel to the short side of the horn. The detector is a diode rectifier mounted in another horn. The DC voltage output of the diode is proportional, to a good approximation, to the intensity of the input electric field. We will use the DMM to measure the voltage.

At some point you may study the operation of the klystron, but for now we will treat it as a "black box" that produces microwaves when appropriate DC voltages are applied to it. Note that the necessary supply voltages are lethal, so you must be sure that the "DC on" switch is in "standby" position before changing any wiring. Check that the klystron is connected as specified in the table, and turn on the AC power switch. Allow about a minute for the tube filament to warm up, and be sure that both high voltage knobs are fully counterclockwise (lowest voltage setting). Set the meter switch to read B+ voltage, and turn on the high voltage. Increase the B+ to 150 V on the built-in meter. You will see a small current, less than 20 mA, on the current meter. Now point the transmitter and receiver horns at each other, with their short sides vertical. Switch the meter to read the C- voltage, and slowly increase it, watching the DMM reading. The klystron can oscillate in several modes, so you will see the power output change with C- voltage as suggested in Fig. 8-5. Use the C- voltage to maximize the output, but stay below 75 V. Once you find a good setting for the power supply, do not disturb it for the rest of the experiment.

As a first experiment, place your hand between the horns, and note that you absorb significantly. (The power level is too low to do any damage to your hand, but corneas are more vulnerable. Do not look into the transmitter horn when power is on.) Try the same experiment with a piece of metal. Next, set the transmitter and receiver next to each other, pointing in roughly the same direction. You should be able to use the metal plate or your hand to reflect microwaves from transmitter to receiver. These exercises should convince you that you will need to keep metallic objects and your body out of the beam, to avoid disturbing the waves.

The microwave horns produce a broadly divergent beam, as you can easily show for yourself by rotating the transmitter horn so that the beam sweeps across the receiver. As a result, it is hard to obtain data of the precision that you might with an optical experiment, where the input beam is very well collimated and of small lateral extent. In partial compensation for this

Connections for klystron

wire	terminal
Brown	6.3 V AC (either)
Brown	6.3 V AC (the other one)
Violet	C-
Red	Common (either)
Orange	Common (either)
White	B+

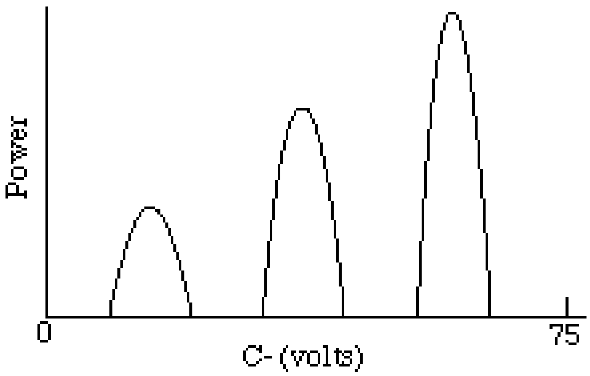

Fig. 8-5 Sketch of klystron power output as a function of C- voltage.

limitation, the microwave beam is fully polarized and much more monochromatic than a conventional light source.

1. Two-slit interferometer

With the preliminaries done, we can proceed to do Young's experiment. Set up the configuration of Fig. 8-6, mounting the three thick metal plates on the plastic base to make two slits, each about 2 cm wide. The horns fit onto two pieces of meter stick, joined by a pivot, so you can vary the angle of observation. To start, place the transmitter about 30 cm from the slits, and the receiver about 50 cm from the slits. Center the pivot under the slits. You should find two or three maxima on each side of the center line as you vary □. Use your measured angles and Eq.⊠-1 to estimate □. Note that the maxima are rather broad, so your determination will not be fantastically accurate. For comparison with later results, it is interesting to estimate the width of the maxima and hence the uncertainty in wavelength.

2. Michelson interferometer

Next, set up a Michelson interferometer as sketched in Fig. 8-7, using a piece of wire mesh as the beam splitter and the large thin metal plates as mirrors. It is convenient to mount the transmitter and one mirror on a meter stick, so that they are automatically aligned. The

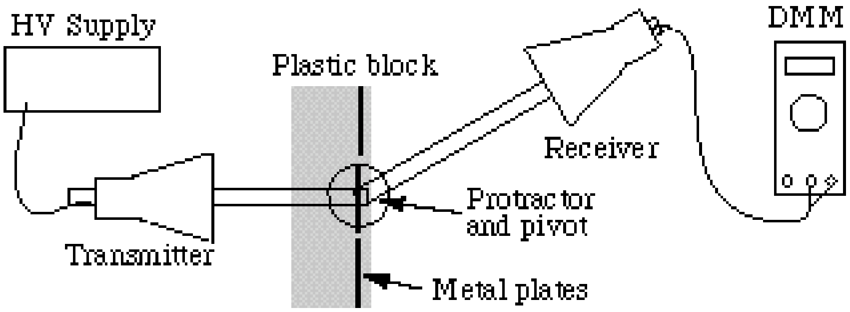

Fig. 8-6 Component layout for two-slit interference measurements.

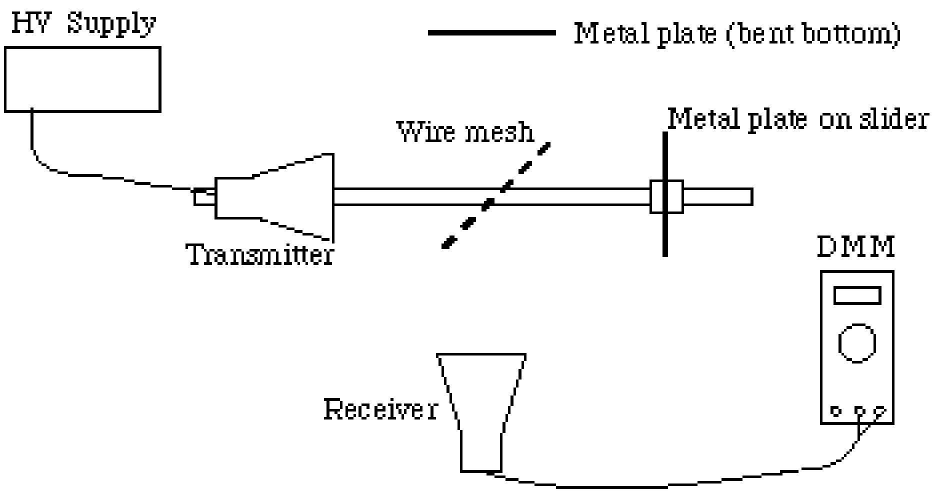

Fig. 8-7 Arrangement for a microwave Michelson interferometer.

combination of the beam splitter, second mirror and receiver can then be independently aligned. By moving the mirror along the meter stick you should see relatively sharp intensity minima and maxima. Use the distance between them to determine the wavelength of the microwaves. Again, it is of interest to estimate the width of the maxima to get an idea of the uncertainty in the wavelength determination.

While the Michelson is set up, we can use it to demonstrate an application of interferometry. Insert the paraffin plate in either arm, and note that the position of the interference minima shifts. Using the amount of shift and the thickness of the plate you can estimate the index of refraction of the paraffin. For best results you will probably need to rotate the wax plate so that reflections from its surfaces do not reach the receiver. If you do this, don't forget to compute the actual path length through the plate.

3. Fabry-Perot interferometer

In order to construct a Fabry-Perot interferometer we must have partially transmitting plates to use as "mirrors". It turns out that a slotted metal sheet does this job quite well. Align the transmitter and receiver horns on the straight piece of meter stick, about 30 cm apart, as in Fig.⊠-8. Place the two slotted plates between the horns with their slots horizontal. Move the plate nearest the receiver along the meter stick to find the maxima. Note the separation between maxima, and also the maximum and minimum transmitted intensity. The maxima are clearest if one plate is kept within about 2 cm of the transmitter, to minimize leakage around it, and the separation *d* is about 10 cm. You will need to keep your hands out of the beam in order to avoid disturbing the measurement. When you have seen the pattern with the slits horizontal, repeat the measurements with the slits vertical. The maxima are very sharp, so you will have to look for

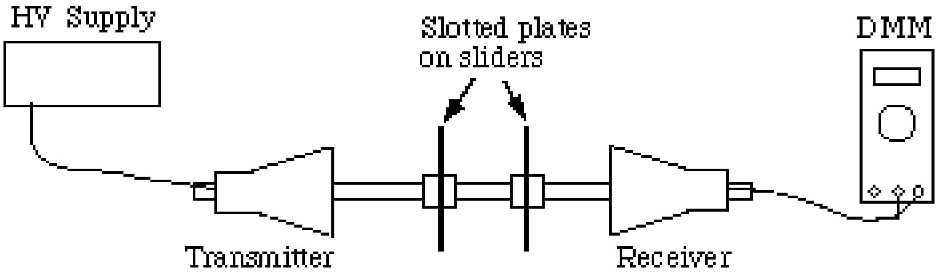

Fig. 8-8 A microwave Fabry-Perot interferometer.

them carefully. Do your results for the two orientations agree qualitatively with Fig. 8-4? Using the measured distance between maxima, calculate λ. Is your result in agreement with what you found from the Young's experiment and the Michelson interferometer? Which result do you think is most precise, and why?

When you are through making measurements, turn down the B+ and C- voltages on the klystron, set the HV switch to standby and then turn off the AC power switch. Leave the klystron connected for the next group of students.

REPORT

Your report should include notes about the relative sharpness of maxima, your measurements of λ, and answers to the questions in the text. Rough sketches of power transmitted vs *d* for the two Fabry-Perot configurations might also be interesting.